TREATMENT OF THERMAL AND RESIDUAL STRESSES IN THE SINTAP DEFECT ASSESSMENT PROCEDURE

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SUMMARY

Within the SINTAP project, Task 4 is addressing thermal and residual stresses. Part of Task 4 deals with production of a compendium of welding residual stresses for a range of weldment geometries. The other part of Task 4 deals with the treatment of these residual stresses, and other secondary stresses, in a defect assessment approach. This latter aspect involves finite-element analysis, experimental work and an examination of the methodology for treating thermal and residual stresses.

This note first describes available methodologies for incorporating thermal and residual stresses in defect assessment procedures. Secondly, the methodologies are compared. Then numerical and experimental results generated during SINTAP are briefly described and discussed in the context of these methodologies. Finally, recommendations are made for treating secondary stresses in the SINTAP procedure and for future work.

1. INTRODUCTION

Experimental work clearly demonstrates that secondary stresses, such as those induced by temperature gradients or the welding process, can have a significant effect on the load carrying capacity of a component containing a flaw [1]. The effect is most pronounced when structural response is essentially elastic; in contrast, residual stresses can have negligible effect when plasticity is widespread. These effects have also been demonstrated numerically by calculating J for structures with and without residual stresses [2].

In view of the above observations, it is important to properly account for secondary stresses in defect assessments. In this note, available and developing methodologies for treating secondary stresses are first described in Section 2. The relationship between the methodologies is then established in Section 3. Section 4 briefly describes the numerical and experimental results generated within the SINTAP project and discusses these in the context of validation of the methodologies described in Section 2. Finally, Section 5 provides recommendations both for the SINTAP procedure and for future work.

2. AVAILABLE METHODOLOGIES

The available methodologies for describing combined primary and secondary stresses may be considered as providing methods for estimating J as a function of the magnitude of the secondary stresses and the magnitude of the primary loads. The former dependence may be assumed to lead to a value J^s for secondary stresses acting alone; the latter dependence may be represented as a dependence on the load ratio L_r. In the failure assessment diagram (FAD) approach, these dependencies are combined through a definition of the parameter K_r such that failure is conceded when $K_r=f(L_r)$, where $f(L_r)$ is the function describing the failure assessment curve which is taken as independent of the secondary loading. In this section, the available methodologies are largely discussed in terms of the definition of K_r but they can equally be expressed in terms of a crack driving force (CDF), J.

2.1 The R6 Method

Within R6 [3], K_r is defined by

$$K_r = (K_I^s + K_I^p)/K_{mat} + \rho \qquad \textbf{(1)}$$

where K_I^s, K_I^p are the linear elastic stress intensity factors for secondary and primary stresses, respectively, K_{mat} is fracture toughness and ρ is a factor which covers interactions between the primary and secondary stresses.

Methods for calculating ρ are given in R6 Appendix 4. The detailed approach in that appendix is to define ρ from

$$\rho = \psi - \phi\,(K_I^s/K_p^s - 1) \qquad \textbf{(2)}$$

where K_p^s is related to the value of J for the secondary loading acting alone, J^s, through

$$K_p^s = \sqrt{E'J^s} \qquad \textbf{(3)}$$

with E' = Young's modulus, E, in plane stress and $E' = E/(1-\nu^2)$ in plane strain, where ν is Poisson's ratio.

The parameters Ψ and ϕ are functions of L_r and the ratio $[K_p^s/(K_I^p/L_r)]$, as tabulated in R6 Appendix 4. Full details are set out in R6 and these are not discussed here. However, there are a number of features of the approach which are worthy of note, as follows:

- For zero primary loads, = 0 and $\phi = 1$ so that eqns (1-3) correspond to failure being conceded when $K_p^s = K_{mat}$ or equivalently when $J^s = K_{mat}^2/E'$.

- There are a number of options for calculating K_p^s in decreasing order of complexity:
    - from eqn (3) using a value of J^s calculated from an inelastic cracked body finite-element analysis;
    - inelastic analysis of the uncracked body;
    - from a plastic zone size correction to K_I^s;
    - when secondary stresses are low compared to the yield stress and elastic follow-up is judged not to be significant K_p^s may simply be replaced by K_I^s.

- The factor is negative for large L_r values ($L_r > 1.05$) and this can lead to negative values of when K_I^s/K_p^s is not significantly less than unity. This then corresponds to mechanical stress relief of the secondary stresses.

- Negative values of can occur even at low mechanical loads if $K_p^s < K_I^s$. This generally corresponds to plastic relaxation of secondary stresses which are elastically in excess of the yield stress.

It is also worth noting that R6 also contains a simplified procedure for evaluating which corresponds to estimating $K_p^s \simeq K_I^s$ and does not allow plastic relaxation of secondary stresses through ensuring 0.

1.2 The V Factor

Within the SINTAP project, Smith[4] has suggested that a factor, V, may be applied to the stress intensity factor, K_I^s, to account for plasticity effects. Then, K_r is defined by

$$K_r = (VK_I^s + K_I^p)/K_{mat} \tag{4}$$

At zero primary loads, V has a value

$$V_o = \sqrt{E'J^s}\,/K_I^s \tag{5}$$

The procedure for using V has not been set out in the same level of detail as that in R6 for the parameter . However, there are a number of features of the approach which are worthy of note, as follows:

- For zero primary loads, eqns(4-5) correspond to failure being conceded when $J^s = K^2_{mat}/E'$ or equivalently when $K^s_p = K_{mat}$.

- The options developed in R6 for estimating K^s_p could equally be used to estimate V_o since $V_o = K^s_p / K^s_I$.

- The factor V becomes less than unity for large L_r values corresponding to mechanical stress relief of the secondary stresses.

- The ratio V/V_o is relatively insensitive to the magnitude of secondary stresses suggesting that a simplified procedure for estimating V could be developed (see Section 3 below).

3. COMPARISON OF   and V METHODS

A review of techniques for treating residual stresses has recently been produced by Smith [5]. This compared the latest approaches in R6, as described in Section 2.1, with earlier methods for estimating   and recommended inclusion of the recent R6 methods in revisions to PD6493 (now BS7910). Therefore, in this section attention is concentrated on a comparison of the methods in Section 2.1 with the V factor approach of Section 2.2. In order to make this comparison, the background to eqn (2) is first set out and this then enables a relationship between V and   to be derived in Section 3.1. Then some numerical results for V and some simplified estimates of V are presented in Section 3.2. Finally, the results of this section are briefly summarised in Section 3.3.

3.1 Derivation of Parameters ρ and V

The background to eqn (2) is set out in [6,7]. In [6], the combined effect of primary and secondary stresses is represented by a reference stress, $_{ref,}$, for the total loading. The value of J for the combined loading is then

$$J = [\sigma_{ref} / f(\sigma_{ref}/\sigma_Y)]^2 \pi \bar{a} / E' \quad \textbf{(6)}$$

where $\bar{a}$ has dimensions of length and is defined by

$$K^p_I = \sigma^p_{ref} (\pi \bar{a})^{1/2} \quad \textbf{(7)}$$

where

$$\sigma^p_{ref} = L_r \sigma_Y \quad \textbf{(8)}$$

For secondary stresses acting alone, the value of J is assumed to be given by J^s as obtained from K^s_p by eqn (3). Following eqn (6), a reference stress , σ^s_{ref} for the secondary loading may then be obtained by solving

$$\sigma^s_{ref} / f(\sigma^s_{ref}/\sigma_Y) = V_o \; K^s_I / (\pi \bar{a})^{1/2} = K^s_p / (\pi \bar{a})^{1/2} \quad \textbf{(9)}$$

In [6], a method is presented for deriving $_{ref}$ in terms of s^{p}_{ref} and s^{s}_{ref}. This leads to a value of J through eqn (6) or equivalently may be used to derive in eqn (1) or V in eqn (4) such that failure within the FAD method at K_r = f(L_r) is consistent with that using a CDF, $J = K^2_{mat}/E'$. Thus, combining eqns (1,6,7,9) leads to

$$\rho = f(L_r) - \frac{f(\sigma_{ref}/\sigma_Y)}{(\sigma_{ref}/\sigma_Y)}\left[L_r + \frac{K^s_I}{K^s_p}\frac{(\sigma^s_{ref}/\sigma_Y)}{f(\sigma^s_{ref}/\sigma_Y)}\right] \qquad \textbf{(10)}$$

or combining eqns (4,6,7,9) gives

$$\frac{V}{V_o} = \left(\frac{\sigma_{ref}}{\sigma^s_{ref}}\right)\frac{f(L_r)f(\sigma^s_{ref}/\sigma_Y)}{f(\sigma_{ref}/\sigma_Y)} - \frac{L_r f(\sigma^s_{ref}/\sigma_Y)}{\sigma^s_{ref}/\sigma_Y} \qquad \textbf{(11)}$$

Equation (10) is identical to eqn (2) with

$$\psi = f(L_r) - \frac{f(\sigma_{ref}/\sigma_Y)}{(\sigma_{ref}/\sigma_Y)}\left[L_r + \frac{(\sigma^s_{ref}/\sigma_Y)}{f(\sigma^s_{ref}/\sigma_Y)}\right] \qquad \textbf{(12)}$$

and

$$\phi = \frac{f(\sigma_{ref}/\sigma_y)}{(\sigma_{ref}/\sigma_y)}\;\frac{(\sigma^s_{ref}/\sigma_y)}{f(\sigma^s_{ref}/\sigma_y)} \qquad \textbf{(13)}$$

as set out in [7]. Values of and ϕ have been obtained from [6] leading to the tables in R6 [3]. With these parameters, eqn (11) can be simply expressed

$$V/V_o = 1 + \psi/\phi \qquad \textbf{(14)}$$

It is apparent that the R6 approach of Section 2.1 can be equally expressed as the V factor method of Section 2.2 provided V is defined by eqn (14). This requires identical information to that in R6: values of K^s_p to define $V_o = K^s_p/K^s_I$ and the parameters ϕ and which are functions of L_r and $(K^s_p L_r/K^p_I)$.

3.2 Numerical and Simplified Estimates of V

Some limits to V may be derived. First, if plasticity effects are assumed to be negligible so that the failure assessment curve is flat (i.e. $f(L_r) \simeq 1$), then response is elastic so that σ_{ref} = $\sigma^p_{ref} + \sigma^s_{ref}$ and V/V_o = 1. Secondly, for secondary loading acting alone, =0 and ϕ = 1 so that again $V=V_o$. Finally, for large values of L_r (>1.05), is negative so that $V/V_o<1$, corresponding to mechanical stress relief.

At intermediate values of L_r some numerical results are shown in Figure 1; these have been evaluated using the values of ϕ and ψ in R6. Note, these values are based on the R6 Option 1 failure assessment curve, but are not expected to be particularly sensitive to the shape of that curve since it has already been noted that $V/V_o = 1$ for a horizontal failure assessment curve. It can be seen that V/V_o increases with increasing L_r initially but only at modest rates. It is also insensitive to the magnitude of the secondary stress, particularly for $L_r>0.9$. This suggests that a simple approximate approach could be developed as an alternative to eqn (14). For example, Figure 1 includes two approximations

$V/V_o = 1.25,$	L_r 0.9	
$V/V_o = 2.78\text{-}1.7\ L_r$	$0.9 < L_r$ 1.4	**(15)**
$V/V_o = 0.4$	$L_r > 1.4$	

$V/V_o = 1.1 + 0.4L_r$	$L_r \le 0.8$	
$V/V_o = 2.78 - 1.7L_r$	$0.8 < L_r \le 1.4$	**(16)**
$V/V_o = 0.4$	$L_r > 1.4$	

Equation (15) bounds the solutions for $K_p^s/(K_I^p/L_r) \le 2$ and uses a constant value for low L_r, similar to the approximate method currently in R6 which uses a constant value of ρ in this region. The plateau value could clearly be made a function of the magnitude of the secondary stress. Equation (16) uses a linear fit at low L_r and bounds the solutions for $K_p^s/(K_I^p/L_r) \le 5$. At values of L_r greater than 0.9, equations (15) and (16) are identical and also correspond to the simple use of ρ = 0 in R6 for $L_r = 1.05$. At higher values of L_r, equations (14-16) correspond to mechanical stress relief of the secondary stresses. The value of 0.4 used for $L_r > 1.4$ in eqns (15) and (16) is simply a bounding value to the numerical results but in practice, full mechanical stress relief (i.e. V = 0) has been observed experimentally [1].

Clearly the approximate approaches of eqns (15) and (16) could be refined based on experimental and finite-element data. Such data are discussed in Section 4.

3.3 Summary of Comparison of ρ and V Methods

In summary, it has been shown that the   and V approaches can be made fully compatible. Each method requires an estimate of the value of J due to secondary loading acting alone: in R6 this is expressed in terms of an effective stress intensity factor K_p^s; with the V approach this is expressed in terms of an initial value V_o. Subsequently,   or V/ V_o depends on the value of L_r and the magnitude of the secondary stresses as expressed in terms of the factor $(K_p^s L_r/K_I^p)$. Tables currently in R6 can be used to evaluate   and V/ V_o. Numerical results for V/V_o suggest that this ratio is relatively insensitive to the magnitude of the secondary stress so that it may be possible to develop simple bounding curves for V/V_o as a function of L_r.

4. VALIDATION OF METHODOLOGIES

With SINTAP Task 4, experimental data have been generated at AEA Technology in tests with and without residual stresses[1]. These covered aluminium plate tests with through cracks and A533B steel plates with surface cracks. Analyses of the aluminium plates using the detailed R6 approach described in Section 2.1 generally gave conservative underestimates of applied load values for various amounts of crack extension. This general conservatism is shown in Fig 2, taken from [1], where the experimental data can be compared to the R6 Option 1 failure assessment curve. It is worth remarking that the R6 analyses are consistent with the secondary stresses having a reduced effect with increasing applied load. This is shown graphically in Fig 3, again from [1], where the ratio of the load carrying capacity of a plate without residual stress to that of a plate with residual stress is plotted as a function of the value of L_r at fracture in the plate with residual stress.

The observations from the aluminium experiments were generally confirmed in the A533B tests but in those tests, R6 assessments significantly underestimated the experimental failure loads. This was attributed to the effects of crack tip constraint for the surface cracks.

The results in [1] were also assessed in terms of the V parameter of Section 2.2. As the V and approaches can be made fully consistent, not surprisingly the experimental data were also conservatively assessed with the V method.

Finite-element results for combined primary and secondary stresses have been generated within SINTAP Task 4 by SAQ[2]. These results showed a rapid decrease in the relative contribution to J of residual stresses, compared to primary stresses, for high values of L_r. Comparisons with R6 predictions, expressed in terms of a crack driving force J, showed R6 to be conservative although the degree of conservatism was dependent on the value of limit load used to derive L_r. As the R6 analyses were conservative, it follows that the V approach of Section 2.2 would also be conservative, although the finite-element results have not been processed in this way.

Finite-element results for plates with under-, over- and even-matched welds have also been produced by TWI[4]. The results for V are shown in Fig 4 which is similar to Fig 1 although reducing to a value less than 0.1 for $L_r > 1.05$. Thus the results again support the method of Section 3.2, with V estimated as depicted in Fig 1. For the overmatched case the results suggest that the methods of Section 3.2 may be over-conservative.

Outside SINTAP, validation of the R6 approach has been performed [7]. As the V approach can be made fully compatible with the approach, Section 3, this may also be taken as validation of the V approach. However, it should be recognised that, apart from analysis of experimental data in [1], there is little experience with the use of V. The choice of method depends largely on ease of use. In view of eqn(14), separate tables for ψ and ϕ would not be required for the V approach; instead a single table for V/V_o could be derived. However, it should be recognised that while the parameter has been shown to be insensitive to the shape of the failure assessment curve, i.e. that ψ and ϕ are not sensitive, it remains to be demonstrated that the ratio ψ/ϕ is also insensitive to the detail of the failure assessment curve. Note, this is more important at high values of L_r (> 1.05), since at lower L_r, failure assessment curves outside R6 Option 1 are expected to lead to values of V/V_o closer to unity than those in Figure 1, as discussed in Section 3.2. Thus, it seems prudent to adopt the R6 approach of Section 2.1 within the SINTAP procedure but to note that the V approach is an alternative developing methodology. This and other recommendations are set out in Section 5 below.

5. RECOMMENDATIONS

On the basis of the review and analysis described above, the following recommendations are made:

(a) The SINTAP procedure should adopt the R6 definition of K_r given in Section 2.1 with allowance for relaxation of secondary stresses due to plasticity being included through the potential for ρ to become negative.

(b) The SINTAP procedure should mention the V approach of Section 2.2 as an alternative developing method and be written in a manner which enables V and V_o to be readily evaluated through their relationships with the parameters K_p^s, ϕ and which are defined in R6.

(c) The calculations leading to V/V_o in Figure 1 should be repeated for different shapes of failure assessment curves, including the curves in the SINTAP procedure.

(d) Finite-element results generated within SINTAP, and elsewhere, should be compared with the theoretical estimates of V/V_o from (c).

(e) The potential for development of a simple bounding curve for V/V_o as a function of L_r, or a table of values, should be considered on the basis of the results of (c) and (d).

6. REFERENCES

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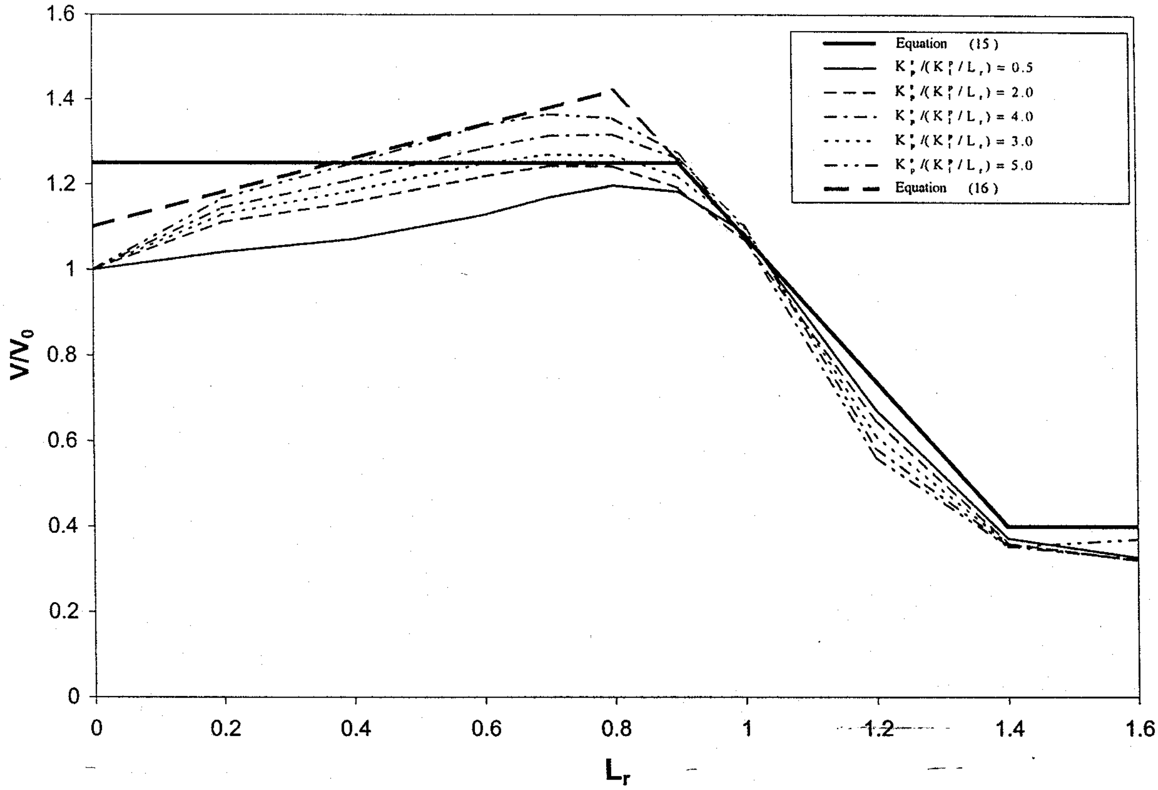

Figure 1 **Numerical Results for V/V_0 from Equation(14) and the R6 Tables for ψ and ϕ, and the Approximations of Equations (15) and (16).**

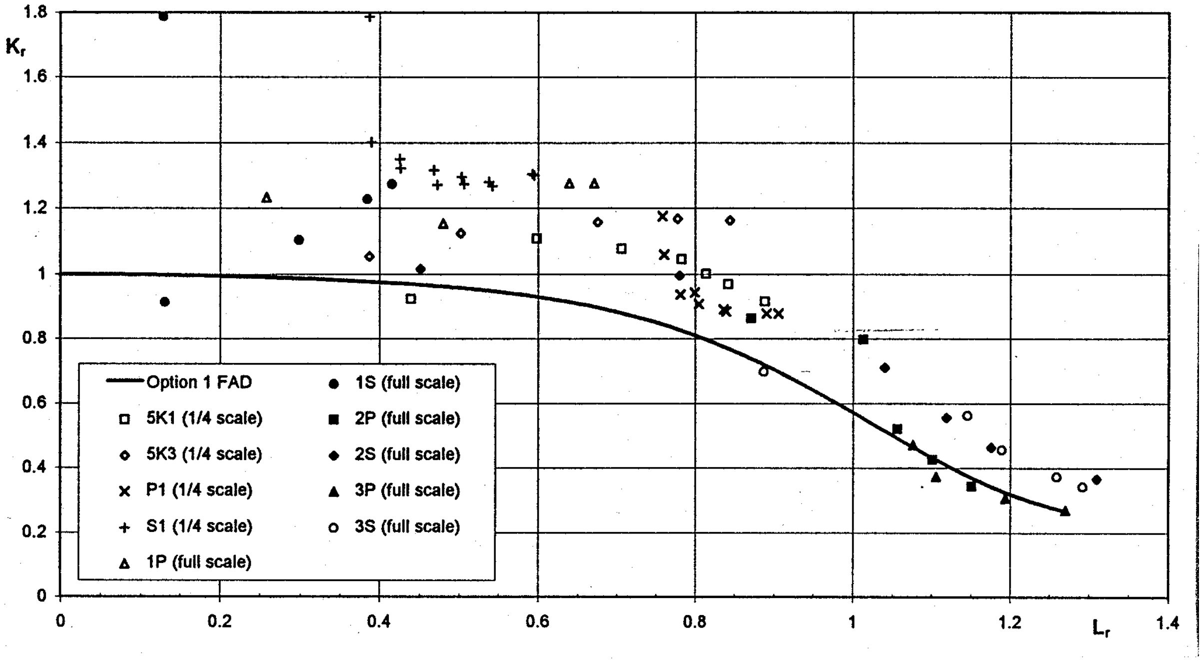

Figure 2 Aluminium Residual Stress Tests Assessed using the R6 Approach of Section 2.1 and Compared to the R6 Option 1 Failure Assessment Curve, from [1]

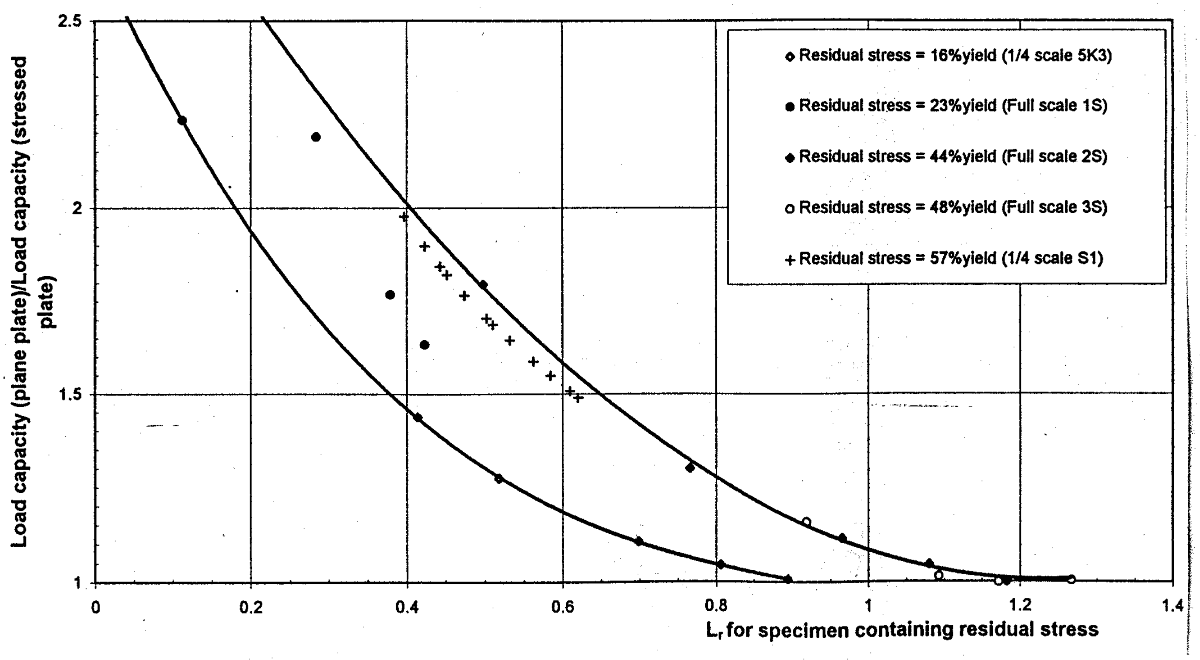

Figure 3 **Ratio of Load Carrying Capacities of Plates with and without Residual Stresses as a function of L_r for Aluminium Tests, from [1]. The lines are simple fits Bounding the Data**

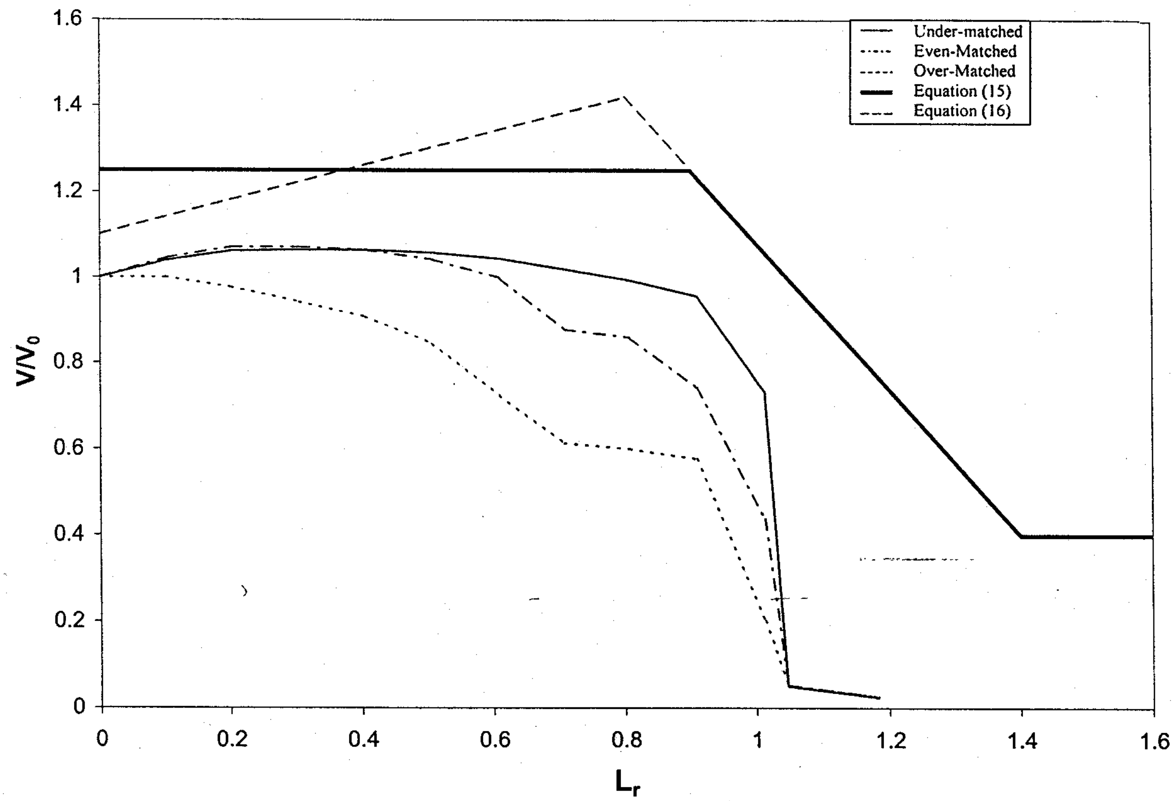

Figure 4 **Comparison of Equations (15) and (16) with Numerical Results from [4] for Under-, Even-, and Over-matched Plates.**