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COMPARISON OF THE PD6493:1991 RHO (ρ) FACTOR WITH FEA RESULTS

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CONTENTS

1. BACKGROUND

Some recent work e.g. (1) has emphasised that the failure assessment diagram (FAD) approach of PD 6493:1991 (2) does not adequately deal with very high secondary stresses (such as exist in as-welded structures). This technical note describes work on a transverse crack under residual stresses with a level equal to the weld metal yield strength. The effect of weld metal mismatch is also studied. More details of the general approach are given in reference 3.

2. THE ρ FACTOR

2.1. THE PURPOSE OF THE ρ FACTOR

The FAD method of performing fitness-for-service (ffs) assessments of welded components is based upon an orthogonal co-ordinate system and a boundary between safe and unsafe regions. A schematic of the Level 3 PD 6493:1991 FAD, for example, is given in Fig.1. The X-axis represents the proximity of primary net section stresses (σ_n) to the flow stress of the material (σ_f). The vertical axis gives the ratio of the elastic stress intensity factor (K_I) to the critical value for the material (K_{mat}). The FAD gives the critical value of K_r, (K_I/K_{mat}), as a function of S_r, (σ_n/σ_f). The shape of the FAD defines the enhanced crack driving force due to crack tip plasticity at higher levels of S_r, but does not allow for crack tip plasticity due to secondary stresses as these do not contribute to σ_n. The increase in driving force due to crack tip plasticity because of secondary stresses is implemented through the so-called rho factor (ρ) which gives $K_r = [(K_I/K_{mat}) + \rho]$.

2.2. CALCULATION OF THE ρ FACTOR FOR YIELD MAGNITUDE WELDING RESIDUAL STRESSES USING PD 6493:1991

PD 6493:1991 describes how to calculate the ρ factor as a function of the primary stress, $P_m + P_b$, the secondary stress, $Q_m + Q_b$, and the yield stress of the material, σ_y (see Fig.12 of (2)). The equations for the calculation of ρ where the initial residual stresses are membrane only (i.e. $Q_b = 0$) and of yield magnitude are given below.

The equations are based upon the factor χ (see Fig.12 of reference 2). Where:

$$\chi = \frac{(Y\sigma)_s}{(Y\sigma)_p}\left\{\frac{\sigma_n}{\sigma_y}\right\}$$

For a small crack loaded by membrane primary and secondary stresses P_m and Q_m it is reasonable to assume that $(Y\sigma)_s/(Y\sigma)_p = Q_m/P_m$ and $P_m \cong \sigma_n$, so $\chi = Q_m/\sigma_y$.

Mechanical reduction of residual stresses due to plastic yielding under high primary and secondary stresses is given by the following expression:

$$Q_m = \text{Lowest of } \left\{ \sigma_y \;\; or \;\; \left(1.4 - \frac{\sigma_n}{\sigma_f}\right)\sigma_y \right\}$$

i.e.

$$Q_m = \text{Lowest of } \left\{ \sigma_y \;\; or \;\; (1.4 - S_r)\sigma_y \right\}$$

The ρ factor for the above assumed conditions is expressed in four regimes by (2). These are given below:

Regime 1 **$S_r < 0.4$**

$\sigma_s = \sigma_y$
$\chi = 1.0$
$\rho = \rho_1 = 0.093$

Regime II **$0.4 < S_r < 0.67$**

$\sigma_s = (1.4 - S_r)\,\sigma_y$
$\chi = (1.4 - S_r)$
$\rho_1 = 0.1\,\chi^{0.714} - 0.007\,\chi^2 + 0.00003\,\chi^5$
$\rho = \rho_1$

Regime III **$0.67 < Sr < 0.875$**

$\sigma_s = (1.4 - S_r)\,\sigma_y$
$\chi = (1.4 - S_r)$
$\rho_1 = 0.1\,\chi^{0.714} - 0.007\,\chi^2 + 0.0003\,\chi^5$
$\rho = 4\,\rho_1\,(1.05 - 1.2\,S_r)$

Regime IV **$S_r > 0.875$**

$\sigma_s = (1.4 - S_r)\,\sigma_y$
$\chi = (1.4 - S_r)$
$\rho = 0$

2.3. CALCULATION OF THE ρ FACTOR FOR YIELD MAGNITUDE RESIDUAL STRESSES USING FEA

Geometry and Mesh Details

The geometry modelled was a butt weld between two flat plates. The plates were each 1m by 2m and the weld was between the 2m edges. This produced a welded plate 2m square (Fig.2). The weld metal was assumed to be a uniform 30mm width. A through-thickness crack of length, 2a, equal to 15mm, was located at the centre of the weld length. The crack was transverse to the weld axis (Fig.2).

The model was of uniform thickness. Two-dimensional, generalised plane strain was assumed. The geometry and loading had two planes of symmetry, so only one quarter was modelled. The model is shown in Fig.3. All elements were ABAQUS type CCPE10H (4). This is a 10-noded bi-quadratic element.

Material Properties

Stress-strain curves for the parent material and the overmatched weld metal were obtained from test data for a carbon manganese pipe and a girth weld, respectively (Fig.4). The stress-strain curve for the undermatched weld was derived from the overmatched weld data by reduction of the yield strength. The parent material stress-strain curve was fixed for all analyses. This had a yield strength of 300N/mm^2. The weld metal stress-strain curves had yield strengths of either 30% above or 30% below the parent value (390N/mm^2 or 210N/mm^2, respectively).

The assumed elastic properties were constant for all analyses: elastic modulus of 207000N/mm^2 and Poisson's ratio of 0.3. A small strain, small displacement finite element formulation was used.

Loading and Boundary Conditions

The mesh is shown in Fig.3. Displacements normal to the symmetry planes were suppressed, except on the face of the crack. The external loading was displacement controlled. The loaded face remained planar throughout each analysis.

The model assumed generalised plane strain conditions. This could allow a uniform out-of-plane displacement and two uniform in-plane rotations for all nodes in the model. The in-plane rotations, however, were suppressed, but the out-of-plane displacement was left free.

Residual Stresses

A residual stress distribution was created via a non-uniform temperature field which produced a peak axial residual stress at the weld centre and a smaller balancing compressive stress in the plates on either side (Fig.5). This static temperature field should not be confused with the transient welding temperature fields - it is an artificial means of creating a balanced residual stress field.

Calculation of the ρ factor

This work has used FEA to calculate the driving force arising from primary and secondary stress separately and together. These results have been used to calculate the effective ρ factor. The method (given below) is based upon the J contour integral (J) which can be used instead of K_I and K_{mat} in (2) (see Section 8.4.4; i.e. $J_e = \frac{K_I^2(1-\nu^2)}{E}$ and $J_{mat} = \frac{K_{mat}^2(1-\nu^2)}{E}$).

Values of J for different levels of applied loading and different modelling assumptions are given in the table below

Table1

Modelling Method	Applied Load	Nomenclature for J
Elastic	P_m	J_e^P
Elasto-plastic	P_m	J_{ep}^P
Elastic	$P_m + Q_m$ Q_m= lower of $\{\sigma_y$ or $(1.4 - S_r)\,\sigma_y\}$*	J_e^{p+s}
Elasto-plastic	$P_m + Q_m$	J_{ep}^{p+s}

*This arises from the PD 6493:1991 expression for mechanical relief of secondary stresses.

The definition of K_r under primary and secondary stresses is as follows:

$$K_r = \frac{K_I^P + K_I^S}{K_{mat}} + \rho = \sqrt{\frac{J_e^{P+S}}{J_{mat}}} + \rho \qquad [1]$$

Under primary loading only, this reduces to:

$$K_r = \frac{K_I^P}{K_{mat}} = \sqrt{\frac{J_e^P}{J_{mat}}} \qquad [2]$$

The critical line on the Level 3 FAD can be written as $K_r = f(S_r)$. The critical line was derived using analytical techniques. An equivalent critical curve for primary loading only can be generated from the FEA by putting $J_{mat} = J_{ep}^P$ in Eq.[2]:

$$K_r^{FEA} = \sqrt{\frac{J_e^p}{J_{ep}^p}} \qquad [3]$$

The error in the Level 2 FAD, e_p, can therefore be determined.

$$e_p = f(S_r) - K_r^{FEA} = f(S_r) - \sqrt{\frac{J_e^p}{J_{ep}^p}} \qquad [4]$$

The rho factor is also determined from the appropriate FAD line. Rho is the additional plasticity correction that is needed for secondary stresses because f(Sr) accommodates plasticity correction for primary stresses only. The effective factor, ρ_{eff}, could therefore be calculated from $f(S_r)$ and finite element results, viz:

$$\rho_{eff} = \left\{ f(S_r) - \sqrt{\frac{J_e^{p+s}}{J_{ep}^{p+s}}} \right\} \qquad [5]$$

2.4. A Proposed Alternative (The V Factor)

PD 6493:1991 has separate approaches for secondary stress crack tip plasticity correction (ρ) and mechanical stress relief of secondary stresses. These allow the user to identify how residual stresses contribute to the ffs calculations. The complexity of ρ, however, means that it is difficult to understand its physical significance. In addition, the ρ factor (see Section 3 below and reference (1)) is not conservative for yield magnitude residual stresses. An alternative approach is proposed. It uses the parameter V, which combines the mechanical relief of residual stress and the plasticity correction, ρ into a single factor which is a function of load (e.g. S_r). The FAD parameter K_r is calculated using V as follows:

$$K_r = \left\{ \frac{K_I^p + V K_{I,o}^s}{K_{mat}} \right\} \qquad [6]$$

With $K_{I,o}^s = (Y\sigma)_s \; (\sigma_n = 0) \; \sqrt{\pi a}$ [7]

Using the nomenclature of Table in Section 2.3, V can be calculated as follows:

$$V = \sqrt{\frac{J_e^p}{J_e^s}} \cdot \sqrt{\frac{J_e^{p+s}}{J_{ep}^{p+s}}} - \sqrt{\frac{J_e^p}{J_e^s}} \qquad [8]$$

3. RESULTS

Curves of ρ_{eff} versus L_r are given in Fig.6 and for the same conditions the parameter V is given in Fig.7. Figure 6 shows that the PD 6493:1991 expressions produce a good accuracy for ρ for L_r above 0.6. Below this level of loading the finite element values for ρ are sensitive to the mismatch conditions. At L_r of zero, all the models give higher values of ρ than PD6493:1991, showing the expressions for ρ in the document are unconservative. This lack of conservatism for ρ persists up to L_r of 0.5 for the overmatched weld, whereas for the undermatch weld the ρ factor drops below the PD 6493:1991 value at L_r of 0.2.

The error in the FAD determined from the finite element analysis (e_p) is plotted against L_r in Fig.8. The FAD for primary loading alone is safe in each case, except for a small portion of the undermatch loading path. The value of e_p is very small for values of L_r below 0.3. This means that the error in ρ makes the PD 6493:1991 FAD unconservative for high levels of residual stresses and low primary loads.

Further work at TWI will concentrate on other geometries and realistic residual stress patterns to determine the extent to which this non conservatism exists. The V parameter will also be generated for a wide range of geometries to find whether a suitably generalised, accurate and simple expression for V can be found.

Future work at TWI in this task will also study the effect of secondary stresses on crack tip constraint.

4. REFERENCES

1 Finch D M and Burdekin F M: 'Finite element validation studies of the revised PD 6493/CEGB R6. Part 1 - Failure assessment methodologies applied to welded flat plates and pipe/plate joints', Int J Pres Vess and Piping 49 (1992), 187-211.

2 British Standard Institution: 'Guidance on some methods for assessing the acceptability of flaws in fusion welded structures'. PD 6493:1991.

3 Smith S D, Pisarski H G and Leggatt R H: 'Influence of residual stresses on crack driving force for weld metal with under and overmatching yield strength'. Mismatch of welds, ESIS publication 17, 1993, 573-587.

4 ABAQUS/Standard User's Manual Volume II Version 5.6, Hibbitt, Karlsson and Seresen, Inc. 1996 (14.1.3-1 to 14.1.3-14).

TWI ENDORSEMENT

This work has been carried out in accordance with TWI's QA Procedures.

Project Leader .... Head of Department ....
(or delegate)

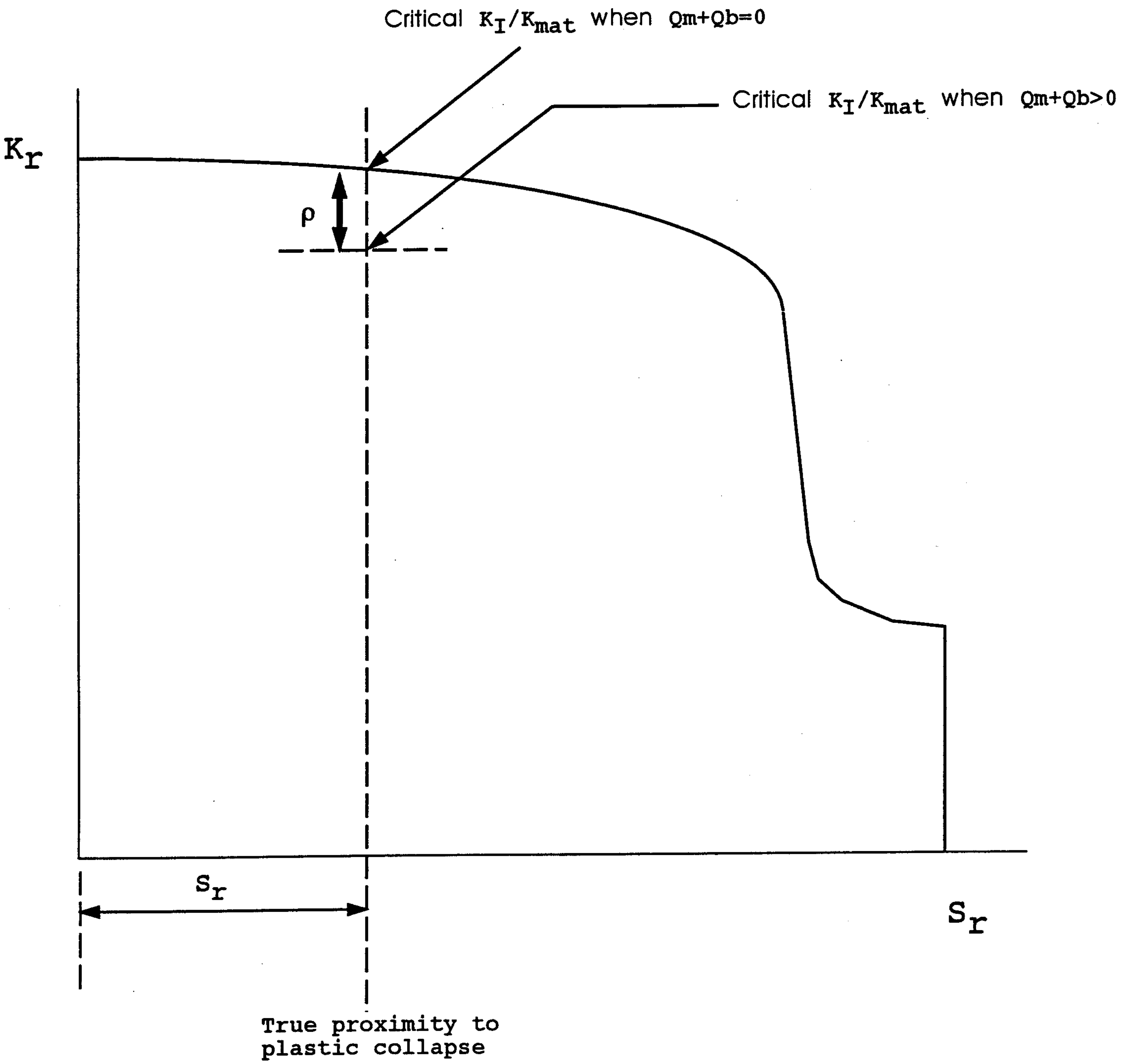
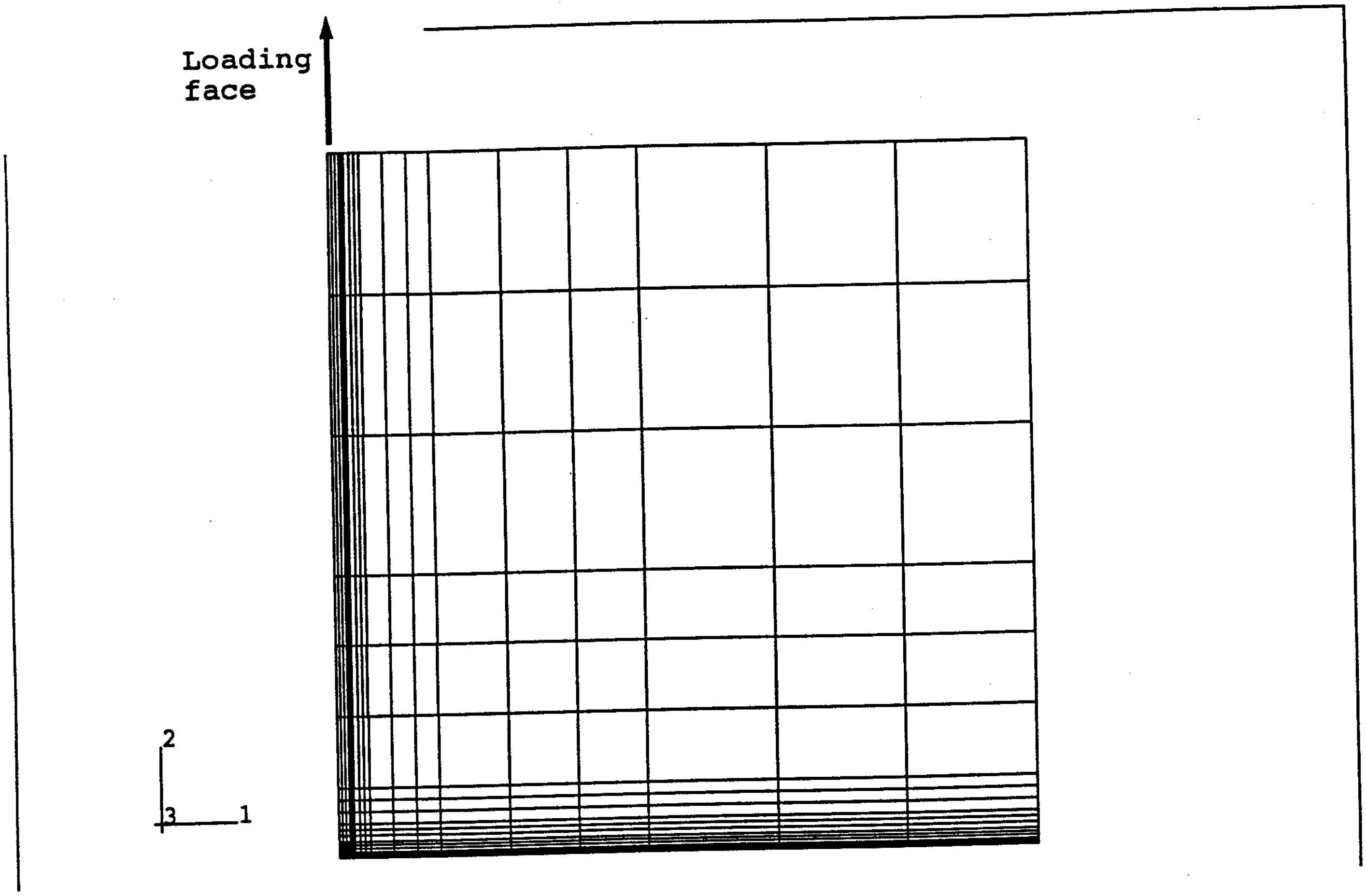


Fig 1 Schematic of the PD6493:1991 Level 3 FAD showing the use of the ρ factor for residual stress plastcity correction.



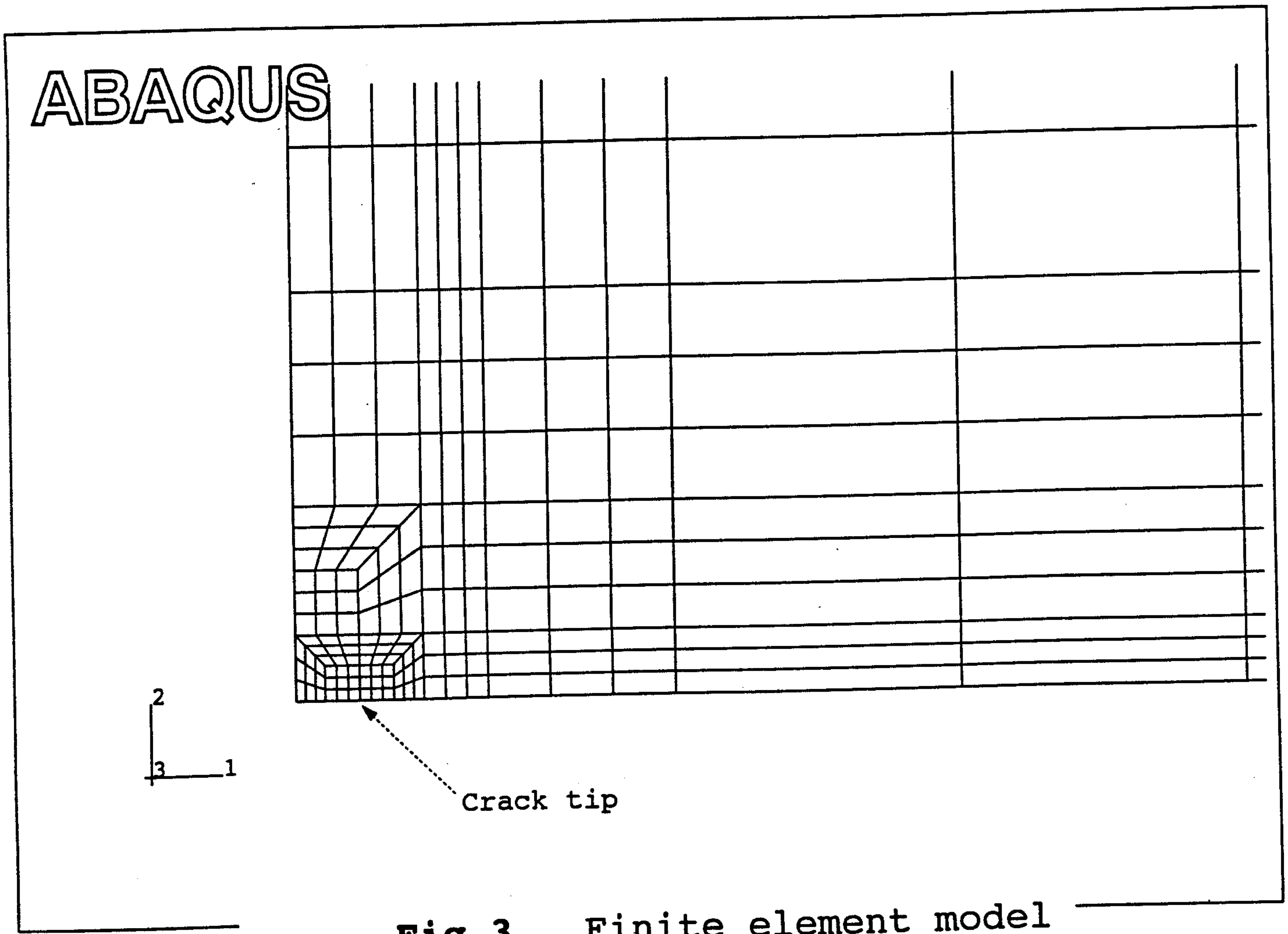


Fig 3 Finite element model

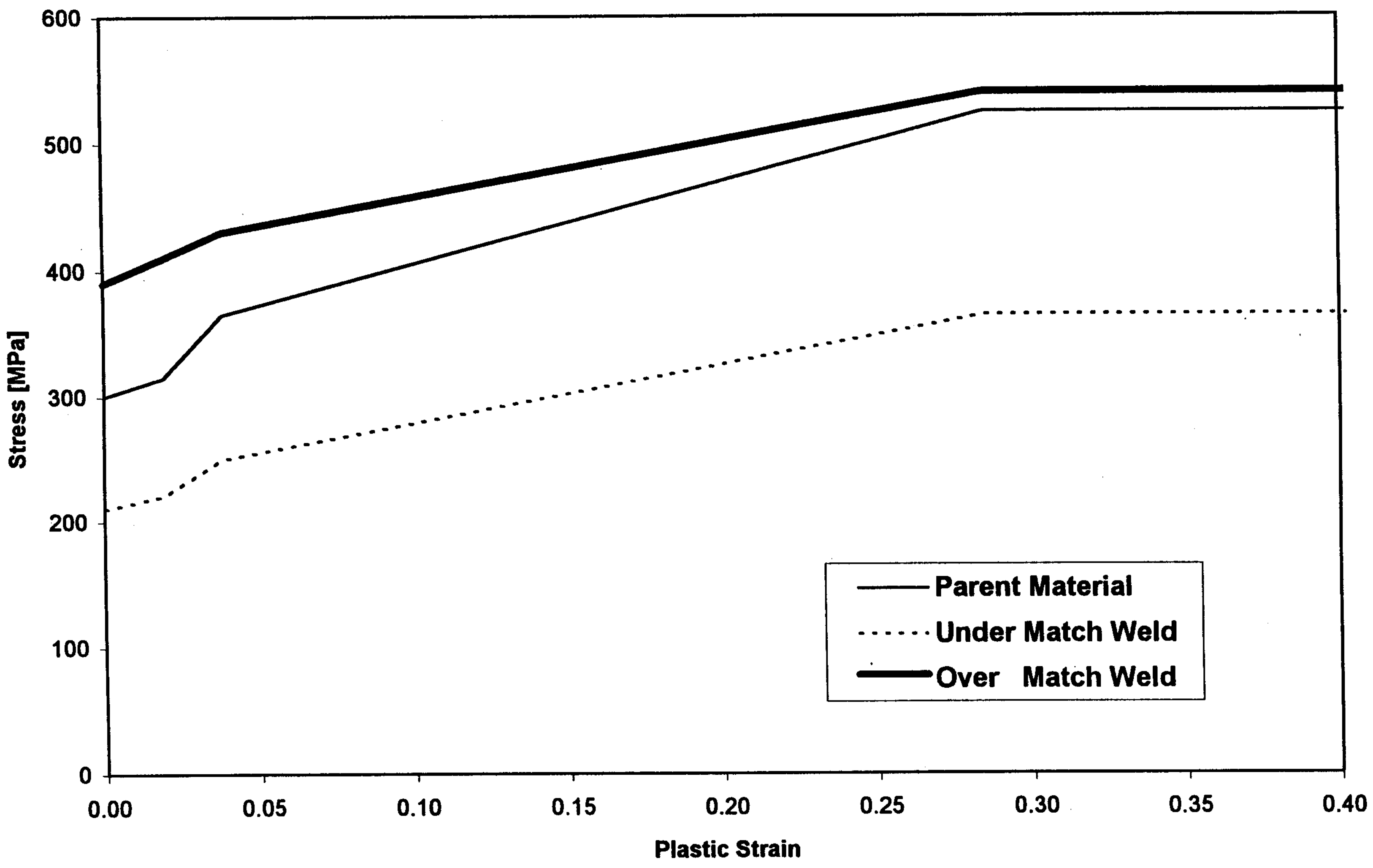


Fig 4 Assumed stress versus plastic strain curves for the weld and parent materials

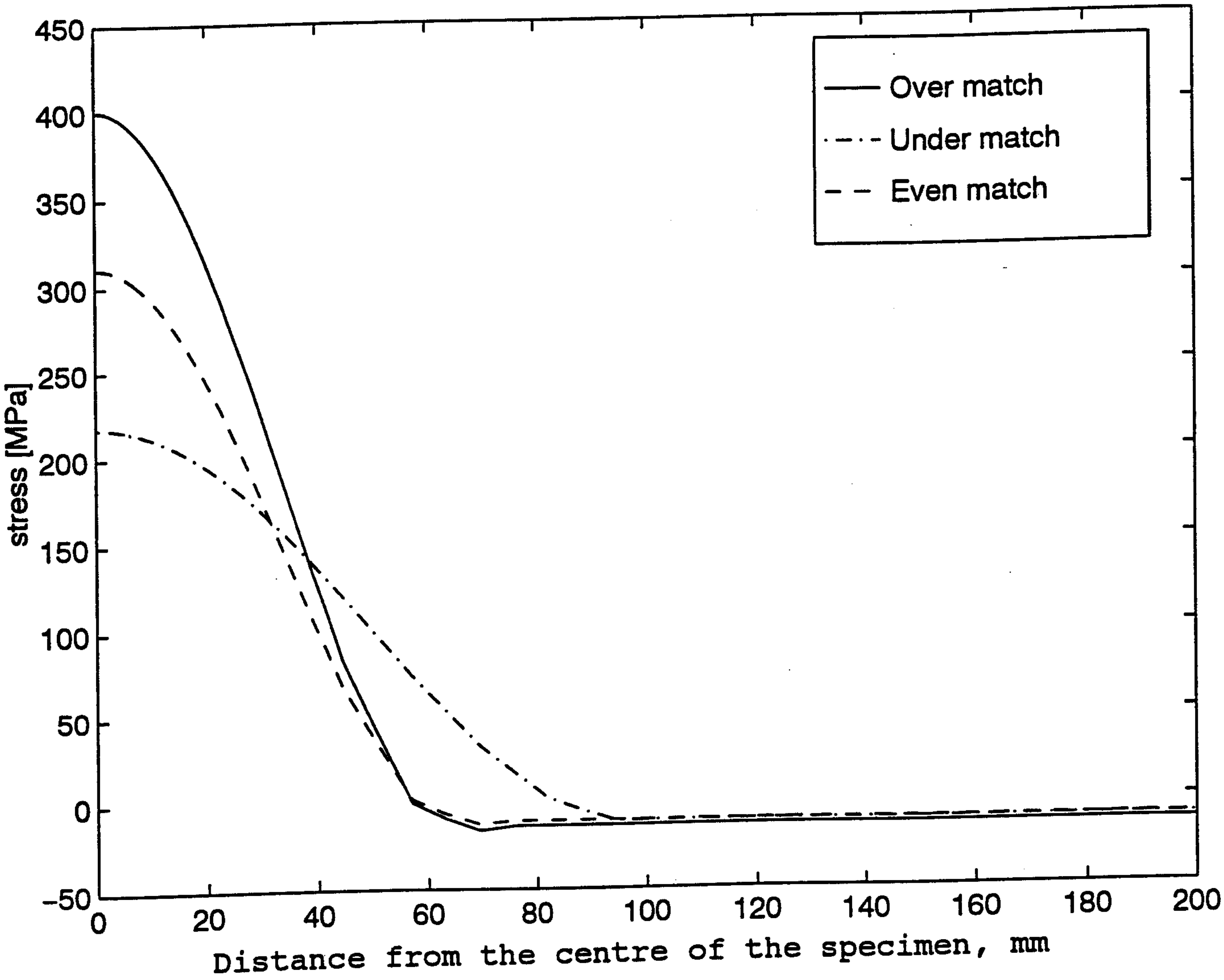


Fig 5 Residual stress distributions in uncracked specimens.

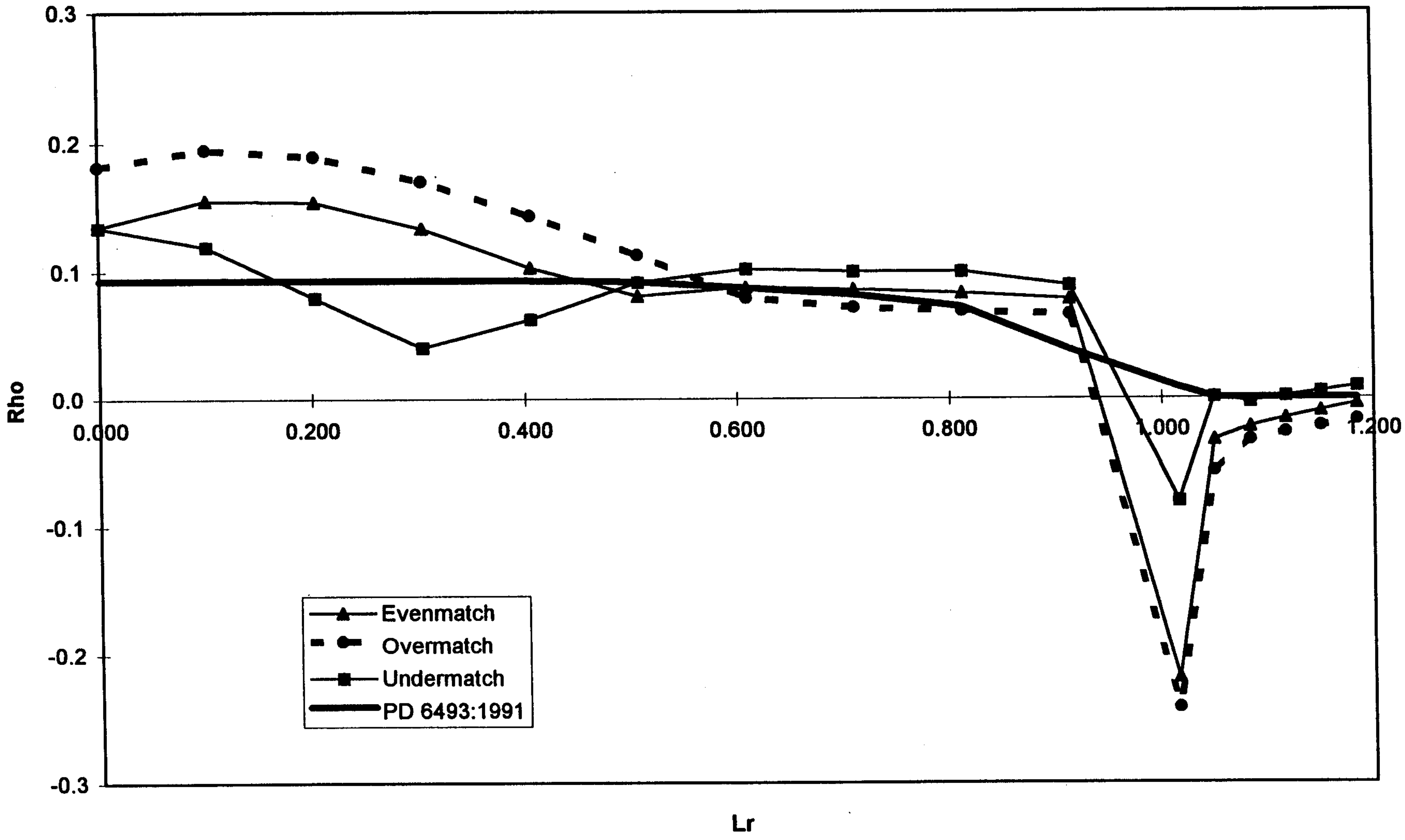


Fig 6 Use of Level 3 PD FAD (Eq 24, pp24) and FEA to calculate rho

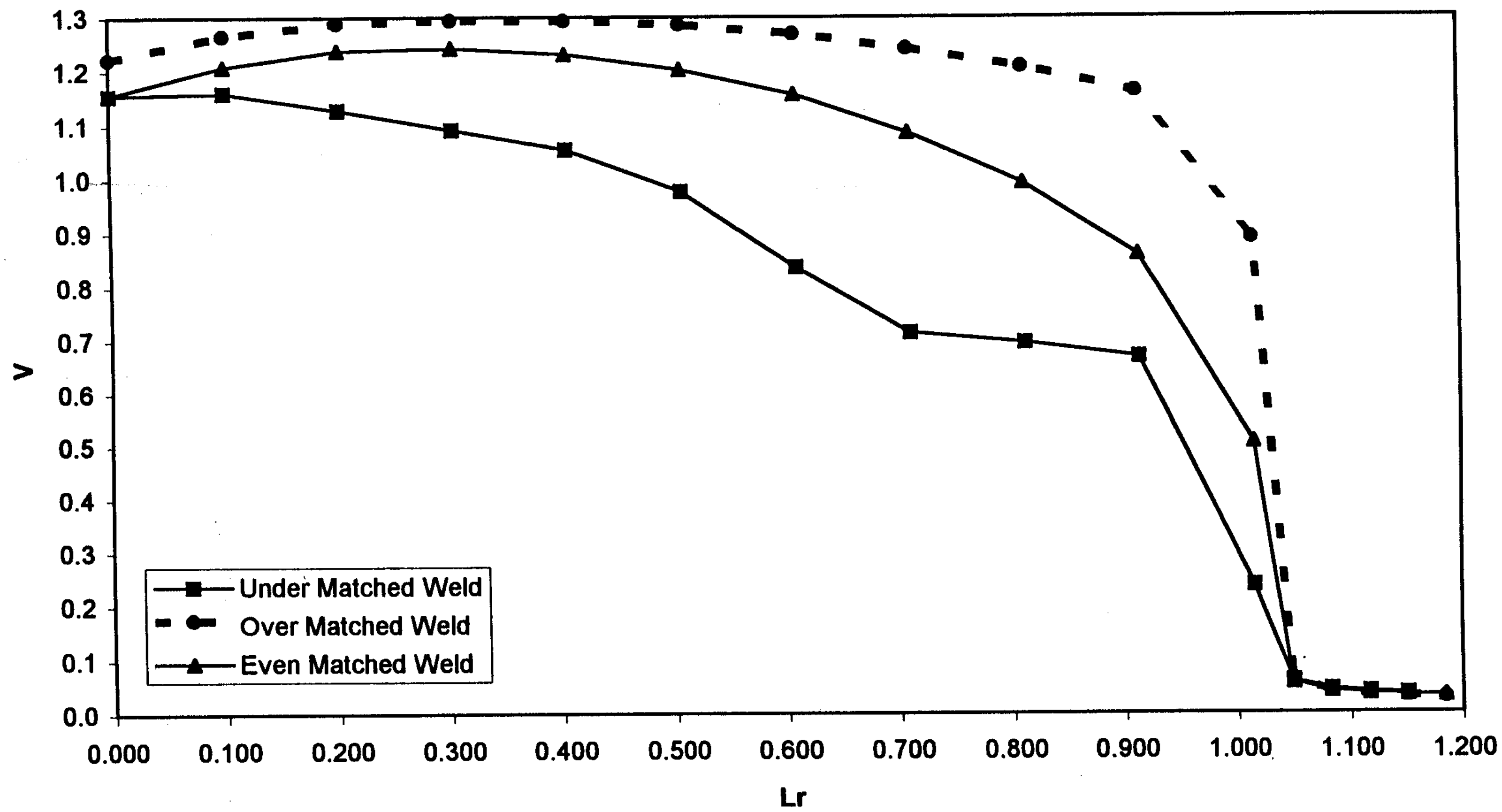


Fig 7 Finite Element results for the parameter, V

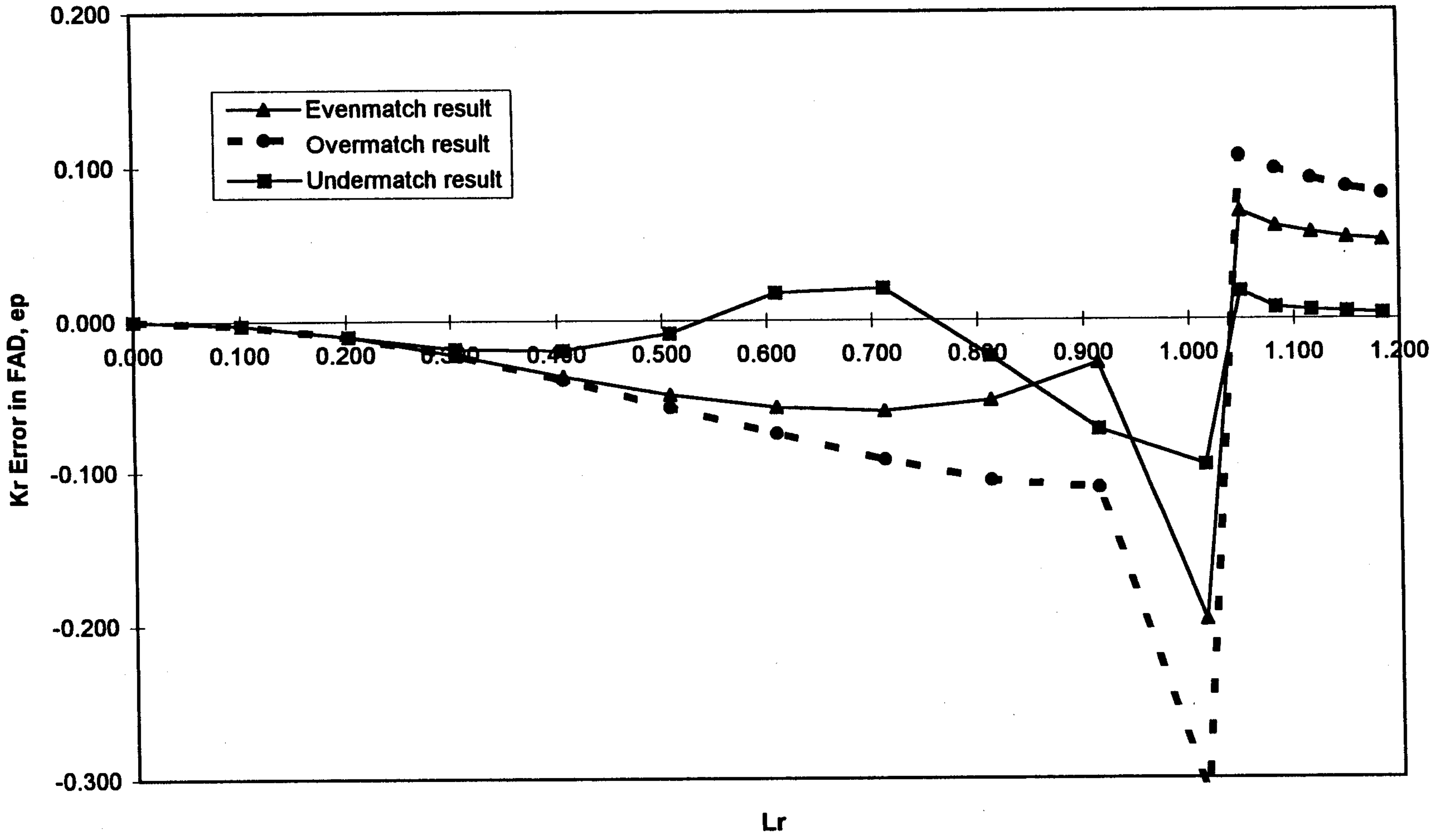


Fig 8 Error in Level 3 FAD from FEA