Failure Assessment Diagram

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Crack Driving Force Diagram

COMPATIBILITY

UNIVERSITY OF CANTABRIA

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INTRODUCTION

This Report summarises the University of Cantabria's proposal for the assessment methodologies unification within the framework of the SINTAP Procedure.

This work has been carried out based on former studies developed by the University of Helsinki [1] and GKSS [2]. Initially, a draft report was produced (not circulated), but further comments from GKSS [3] have added new ideas that have been included in this version. All these analyses have been integrated into this work and amplified by our own developments.

We have incorporated this research into the initial structure of SINTAP (suggestion B). So, "basic", "assumed N" and "measured N" levels have been defined, in which the desired FAD-CDFD compatibility has been obtained. Some comments about "material & geometry specific" level are also presented.

Therefore, all these three levels could be run by the user using either a Failure Assessment Diagram or a Crack Driving Force Diagram. It does not matter which graphic representation is used because both methodologies have been found to be compatible.

Two analysis paragraphs are incorporated at each level studied. The first one, describing the Failure Assessment Line (FAL) for a FAD; and the second one, defining the applied J-integral for a component as a Crack Driving Force (CDF) for a CDFD.

LEVEL 1

BASIC

1.1 OBJECTIVE

- Default curves with no previous knowledge of the material and component.

1.2 OPTIONS

- A: Continuously hardening materials, without yield plateau.
- B: Material with yield plateau.

1.3 ASSUMPTIONS

- A: The materials harden from the starting point following a piece-wise power law with an exponent of 0.3.
- B: The material's behaviour is assumed to be linear-elastic up to σ_y.
- B: From σ_y on, a lower bound material is considered consisting of a not-hardening or elastic-perfectly plastic material.

1.4 RELEVANT EQUATIONS

$$K_r = \left[\frac{E\varepsilon_{ref}}{L_r \sigma_y} + \frac{L_r^3 \sigma_y}{2E\varepsilon_{ref}} \right]^{-1/2} \quad [4] \qquad (1.1)$$

$$\sigma = \sigma_y \left(\frac{\varepsilon}{\varepsilon_y} \right)^{0.3} \quad \text{(opt. A)} \qquad (1.2a)$$

$$\begin{cases} \sigma = E\varepsilon & \text{for } \sigma \leq \sigma_y,\ \varepsilon \leq \varepsilon_y \\ \sigma = \sigma_y & \text{for } \varepsilon > \varepsilon_y \end{cases} \quad \text{(opt. B)} \qquad (1.2b)$$

1.5 ANALYSIS

- **FAL**

∗ **Option A**

GKSS [3] has developed curves (Eq. (2.3a)) which depend exclusively on the strain hardening coefficient (N). This analysis has been performed experimentally from the initial idea of R6 FAD1. We propose that the **"BASIC"** Failure Assessment Line for continuously hardening materials could be the curve calculated for N=0.3 with a little correction, i.e. change the first factor of Equation (2.3a) by the first section of Equation (1.3b), thus giving Equation (1.3a).

$$K_r = \left[1 + \frac{L_r^2}{2}\right]^{-1/2} \left[0.3 + 0.7\exp(-0.6 L_r^6)\right] \qquad \text{(opt. A)} \qquad (1.3a)$$

The reason of that change can be justified because both factors are graphically very similar, but the second one can be obtained from a mechanical point of view. Figure 1 shows the comparison between FALs developed using both factors.

∗ **Option B**

Inserting Equation (1.2b) into Equation (1.1) we get the **"BASIC"** Failure Assessment Line expression (Figure 2) for a material exhibiting a yield plateau.

$$\begin{cases} K_r = \left[1 + \frac{L_r^2}{2}\right]^{-1/2} & \text{for } L_r \le 1 \\ K_r = 0 & \text{for } L_r > 1 \end{cases} \qquad \text{(opt. B)} \qquad (1.3b)$$

Figure 3 presents all options of the BASIC Failure Assessment Line: that defined by GKSS and the ones proposed by UC represented by Equations (1.3a) and (1.3b).

- **CDF**

Making use of "Annex 1: Transformation FAL → CDF. Equation (A1.6)", we can obtain the CDF equivalent to the **"BASIC"** Failure Assessment Line for both options

$$J = \frac{K_I^2}{E'}\left[1 + \frac{L_r^2}{2}\right]\left[0.3 + 0.7\exp(-0.6 L_r^6)\right]^{-2} \qquad \text{(opt. A)} \qquad (1.4a)$$

$$J = \frac{K_I^2}{E'}\left[1 + \frac{L_r^2}{2}\right] \qquad \text{(opt. B)} \qquad (1.4b)$$

LEVEL 2

ASSUMED N

2.1 OBJECTIVE

- Development of curves through a simplified stress-strain curve model.

2.2 OPTIONS

- A: Continuously hardening materials, without yield plateau.
- B: Material with yield plateau.

2.3 ASSUMPTIONS

- A: The materials harden from the starting point following a piece-wise power law.
- B: The material's behaviour is assumed to be linear-elastic up to σ_y.
- B: There could be a yield plateau.
- B: From σ_y on, the stress-strain curve can be modelled as a piece-wise power law.

2.4 RELEVANT EQUATIONS

$$K_r = \left[\frac{E\varepsilon_{ref}}{L_r \sigma_y} + \frac{L_r^3 \sigma_y}{2E\varepsilon_{ref}} \right]^{-1/2} \qquad (2.1)$$

$$\sigma = \sigma_y \left(\frac{\varepsilon}{\varepsilon_y} \right)^N \qquad \text{(opt. A)} \qquad (2.2a)$$

$$\left[\begin{array}{ll} \sigma = E\varepsilon & \text{for } \sigma \le \sigma_y,\ \varepsilon \le \varepsilon_y \\ \sigma = \sigma_y & \text{for } \varepsilon_y < \varepsilon \le \varepsilon_y + \Delta_\varepsilon \\ \sigma = \sigma_y \left(\dfrac{\varepsilon}{\varepsilon_y + \Delta_\varepsilon} \right)^N & \text{for } \sigma > \sigma_y,\ \varepsilon > \varepsilon_y + \Delta_\varepsilon \end{array} \right. \qquad \text{(opt. B))} \qquad (2.2b)$$

2.5 ANALYSIS

- **FAL**

∗ **Option A**

For stresses under σ_y (L_r □ 1), GKSS has developed curves (Eq. (2.3a)) which depend exclusively on the strain hardening coefficient (N). This analysis has been performed experimentally from the initial idea of R6 FAD1. These curves constitute the **"ASSUMED N"** Failure Assessment Line for a continuously hardening material Although the University of Cantabria has tried to obtain them analytically by simplifying Eq. (2.1) for a law that satisfies Eq. (2.2a), the mathematical expressions obtained don't follow the real material behaviour as they grow in the interval from L_r=0 to L_r=1.

$$K_r = \left[1 - \frac{L_r^2}{5}\right]\left[0.3 + 0.7\exp(-2NL_r^6)\right] \qquad \text{(opt. A)} \qquad (2.3a)$$

Once again, UC proposes to change the first factor of Eq. (2.3a) for the first section of Eq. (1.3b), resulting:

$$K_r = \left[1 + \frac{L_r^2}{2}\right]^{-1/2}\left[0.3 + 0.7\exp(-2NL_r^6)\right] \qquad \text{(opt. A)} \qquad (2.4a)$$

For values greater than σ_y (L_r □ 1), GKSS follows "Annex 2: Transformation CDF → FAL", giving:

$$K_r = K_{r(L_r=1)} L_r^{\frac{N-1}{2N}} \qquad \text{(opt. A)} \qquad (2.5a)$$

where the value for K_r at L_r=1 is obtained through Eq. (2.4a).

Figure 4 shows different FALs constructed following Equations (2.4a) and (2.5a) for different strain hardening coefficients.

∗ **Option B**

Inserting the different parts of Eq. (2.2b) into Eq.(2.1) we get the **"ASSUMED N"** Failure Assessment Line expression for a material with yield plateau in three different sections.

First section. Elastic behaviour up to L_r = 1.

$$K_r = \left[1 + \frac{L_r^2}{2}\right]^{-1/2} \quad \text{for } L_r \,□\, 1,\ \sigma \,□\, \sigma_y,\ \varepsilon \,□\, \varepsilon_y \qquad \text{(opt. B)} \qquad (2.3b)$$

Second section. Yield plateau at $L_r = 1$.

$$K_r = \left[\lambda + \frac{1}{2\lambda}\right]^{-1/2} \quad \text{for } L_r = 1,\ \sigma = \sigma_y,\ \varepsilon_y < \varepsilon \le \varepsilon_y + \Delta_\varepsilon \qquad \text{(opt. B)} \qquad (2.4b)$$

where: $$\lambda = 1 + \frac{E}{\sigma_y}\Delta \quad \text{and} \quad \lambda_{max} = 1 + \frac{E}{\sigma_y}\Delta_\varepsilon \qquad (2.5b)$$

being Δ_ε the extension of the yield plateau and $\Delta \le \Delta_\varepsilon$.

Third section. Strain hardening behaviour from $L_r = 1$.

$$K_r = \left[\lambda_{max} L_r^{\frac{1}{N}-1} + \frac{L_r^{3-\frac{1}{N}}}{2\lambda_{max}}\right]^{-1/2} \quad \text{for } L_r > 1,\ \sigma > \sigma_y,\ \varepsilon > \varepsilon_y + \Delta_\varepsilon \qquad (2.6b)$$

This third section (Eq. (2.6b)) could have been developed by means of "Annex 2: Transformation CDF → FAL". In such a case, the same equation that (2.5a) would have been obtained where K_r value for L_r=1 is calculated through Eq. (2.4b) for λ_{max}, thus giving Equation (2.7b).

$$K_r = \left[\lambda_{max} + \frac{1}{2\lambda_{max}}\right]^{-1/2} L_r^{\frac{N-1}{2N}} \quad \text{for } L_r > 1,\ \sigma > \sigma_y,\ \varepsilon > \varepsilon_y + \Delta_\varepsilon \qquad (2.7b)$$

Figure 5 shows the similarities between both options (Equations (2.6b) and (2.7b)) for a series of materials.

Figure 6 shows a FAL developed through this formulation compared to the corresponding R6 FAD2 for a ferritic steel. This material's stress-strain curve is presented in Figure 7.

- **CDF**

Making use of "Annex 1: Transformation FAL → CDF. Equation (A1.6)", we can obtain the CDF equivalent to both options.

$$J = \frac{K_I^2}{E'} \cdot \left[f(L_r)\right]^{-2} \qquad (A1.6)$$

LEVEL 3

MEASURED N

3.1 OBJECTIVE

- Development of curves with the help of the actual true stress-strain curve.

3.2 RELEVANT EQUATIONS

$$K_r = \left[\frac{E\varepsilon_{ref}}{L_r \sigma_y} + \frac{L_r^3 \sigma_y}{2E\varepsilon_{ref}} \right]^{-1/2} \tag{3.1}$$

3.3 ANALYSIS

- **FAL**

Equation (3.1) directly represents the **"MEASURED N"** Failure Assessment Line.

- **CDF**

Making use of "Annex 1: Transformation FAL → CDF. Equation (A1.6)", we can obtain the CDF equivalent to Eq. (3.1):

$$J = \frac{K_I^2}{E'} \left[\frac{E\varepsilon_{ref}}{L_r \sigma_y} + \frac{L_r^3 \sigma_y}{2E\varepsilon_{ref}} \right] \tag{3.2}$$

LEVEL 4

MATERIAL & GEOMETRY SPECIFIC

4.1 OBJECTIVE

- Development of curves for a specific material and component.

4.2 ASSUMPTIONS

- The component is defined through the ETM master-type curve.

4.3 RELEVANT EQUATIONS

$$\frac{J}{J_Y} = \left(\frac{F}{F_Y}\right)^{\frac{1+N}{N}} \quad [5] \qquad (4.1)$$

4.4 ANALYSIS

- **FAL**

The expression of the Failure Assessment Line can be obtained following "Annex 2: Transformation CDF → FAL" and is presented in Equation (A2.8).

$$K_r = K_{r(L_r=1)} \frac{K_I}{K_Y} L_r^{-\frac{1+N}{2N}} = K_{r(L_r=1)} L_r^{\frac{N-1}{2N}} \qquad (A2.8)$$

As can be seen, the Failure Assessment Line is not dependant on the geometry resulting on Level 2 once again. This is so, because the ratio between K_I and K_Y can be substituted by L_r as this ratio is proportional to σ/σ_y on the Failure Assessment Line. Therefore, the effect of the crack and the geometry dissapears.

- **CDF**

The applied J-integral for a component comes directly from the assumption of this section -represented by Equation (4.2)- giving Equation (A2.9).

$$J = J_Y L_r^{\frac{1+N}{N}} \qquad (A2.9)$$

ANNEX 1

TRANSFORMATION FAL→CDF

In a Failure Assessment Diagram, the Failure Assessment Line should follow the equation:

$$K_r^{LINE} = \left(\frac{J_e}{J}\right)^{1/2} \tag{A1.1}$$

Any simplification of this equation which is considered as the Failure Assessment Line on a FAD can be expressed as follows:

$$K_r^{LINE} = f(L_r) \tag{A1.2}$$

The better the function $f(L_r)$ fits Equation (A1.1), the more accurate the assessment is. So, as Equation (A1.2) represents the boundary of the safe area, this can be taken as the mathematical expression of (A1.1). Therefore both equations can be equalled:

$$\left(\frac{J_e}{J}\right)^{1/2} \approx f(L_r) \tag{A1.3}$$

From that, we obtain:

$$J = J_e \cdot [f(L_r)]^{-2} \tag{A1.4}$$

where

$$J_e = \frac{K_I^2}{E'} \tag{A1.5}$$

Inserting Eq. (A1.5) into Eq. (A1.4), gives

$$\mathbf{J = \frac{K_I^2}{E'} \cdot [f(L_r)]^{-2}} \tag{A1.6}$$

which represents the applied J-integral of a component as a function of the stress intensity factor and the Failure Assessment Line considered.

ANNEX 2

TRANSFORMATION CDF→FAL

Following the ETM-master type curve:

$$\frac{J}{J_Y} = \left(\frac{F}{F_Y}\right)^{\frac{1+N}{N}} \qquad \text{(A2.1)}$$

we can obtain the expression of the Failure Assessment Line for L_r □ 1 by means of an easy coordinate change:

$$\frac{F}{F_Y} = L_r \qquad \text{(A2.2)}$$

$$K_r = \left(\frac{J_e}{J}\right)^{1/2} = \left(\frac{K_I^2}{E'J}\right)^{1/2} \qquad \text{(A2.3)}$$

Inserting Eq. (A2.2) into Eq. (A2.1) and then, this one into Eq. (A2.3), gives:

$$K_r = \left(\frac{K_I^2}{E'J_Y}\right)^{1/2} \cdot L_r^{-\frac{1+N}{2N}} \qquad \text{(A2.4)}$$

where J_Y must be calculated at L_r=1 from the Failure Assessment Line expression for L_r □ 1 in order to give continuity to the overall function. If we note that point by:

$$K_{r(L_r=1)} \qquad \text{(A2.5)}$$

we can assure that we must obtain the same value if calculated through Eq. (A2.3)

$$K_{r(L_r=1)} = \left(\frac{K_Y^2}{E'J_Y}\right)^{1/2} \qquad \text{(A2.6)}$$

from that, we can get the J_Y value

$$J_Y = \frac{K_Y^2}{E'(K_{r(L_r=1)})^2} \qquad \text{(A2.7)}$$

Once this value is known, it can be incorporated into Eq. (A2.4) giving:

$$\mathbf{K_r = K_{r(L_r=1)} \frac{K_I}{K_Y} L_r^{-\frac{1+N}{2N}} = K_{r(L_r=1)} L_r^{\frac{N-1}{2N}}} \qquad \textbf{(A2.8)}$$

which is the Failure Assessment Line compatible with a Crack Driving Force defined by:

$$\mathbf{J = J_Y L_r^{\frac{1+N}{N}}} \qquad \textbf{(A2.9)}$$

CONCLUSIONS

In this work, an effort has been done in order to find the compatibility between the different FAD and CDFD levels suggested within the SINTAP Procedure initial structure [6]. In our opinion, this has been done and demonstrated to be possible for all levels except those concerning special circumstances like constraint or mismatch, about which, no analysis has been studied yet.

The scope of this document can be summarised in the following table:

			FAD	CDFD
BASIC	LEVEL 1	Default	√	√
MATERIAL SPECIFIC	LEVEL 2	Assumed N	√	√
	LEVEL 3	Measured N	√	√
MATERIAL & GEOMETRY SPECIFIC	LEVEL 4	CDF → FAL	√ (*)	√
		Special features (constraint, mismatch)	-	-

(*) If a FAD is used, the same Failure Assessment Line that Level 2 is obtained, so the crack growth effect is not considered when evaluating a structure where ductile tearing is allowed. Thus, different Failure Assessment Lines for different crack sizes have to be developed if a more exact and precise evaluation is desired. It is our belief that the assessment becomes too complicated [7]. Therefore, our recommendation is to use a complete Crack Driving Force analysis where the component's applied J-integral is compared to the full J_R curve.

REFERENCES

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[4] Milne, I.; Ainsworth, R.; Dowling, A.R. and Stewart, A.T. "Assessment of the Integrity of Structures Containing Defects", CEGB, Report R/H/R6-Rev.3, 1987.

[5] Schwalbe, K.-H.; Zerbst, U.; Brocks, W.; Cornec, A.; Heerens, J. and Amstutz, H. "The Engineering Treatment Model for Assessing the Significance of Crack-like Defects in Engineering Structures (EFAM ETM 95)", GKSS-Forschungszentrum, 1996.

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[7] Ruiz Ocejo, J.; González-Posada, M.A.; Gutiérrez-Solana, F. and Gorrochategui, I. "Review of Existing Procedures", Subtask 5.1, Report, SINTAP/UC/04, June 1997.

Figure 1
BASIC
Continuously hardening material

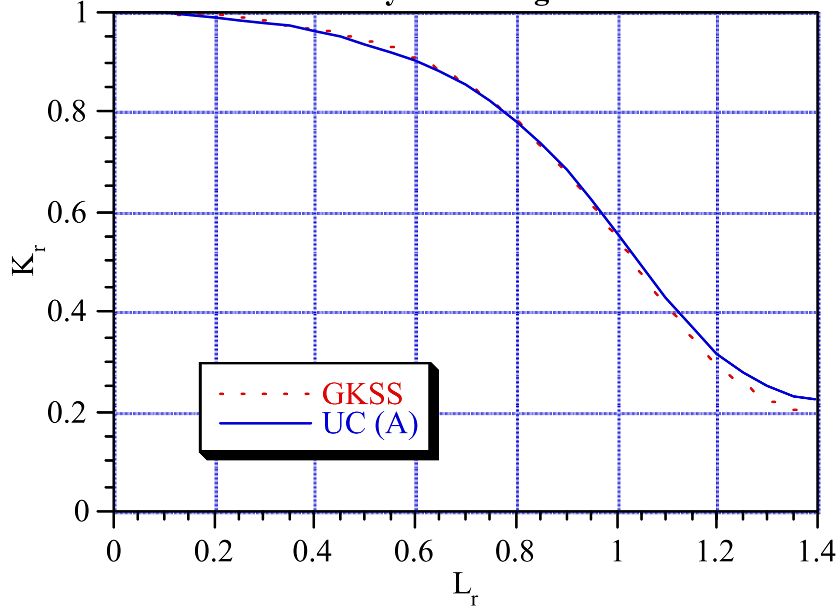
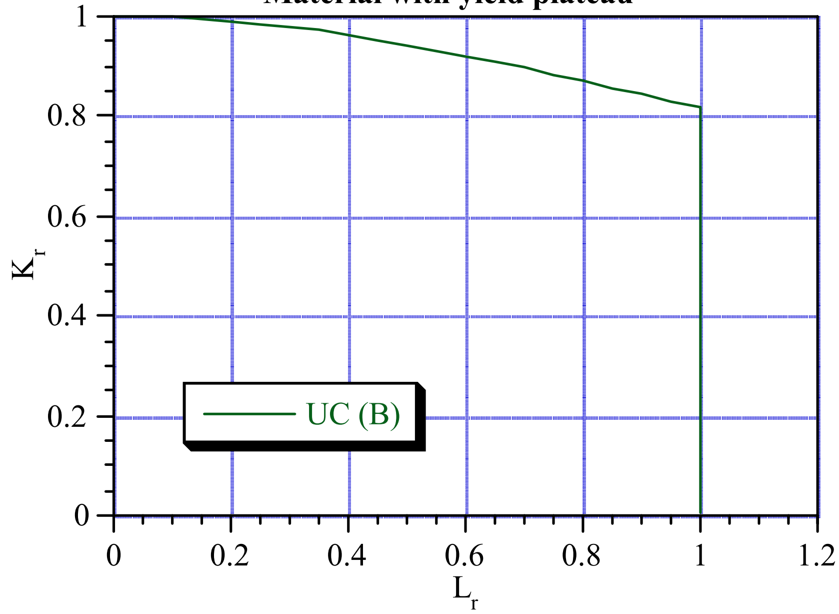


Figure 2
BASIC
Material with yield plateau



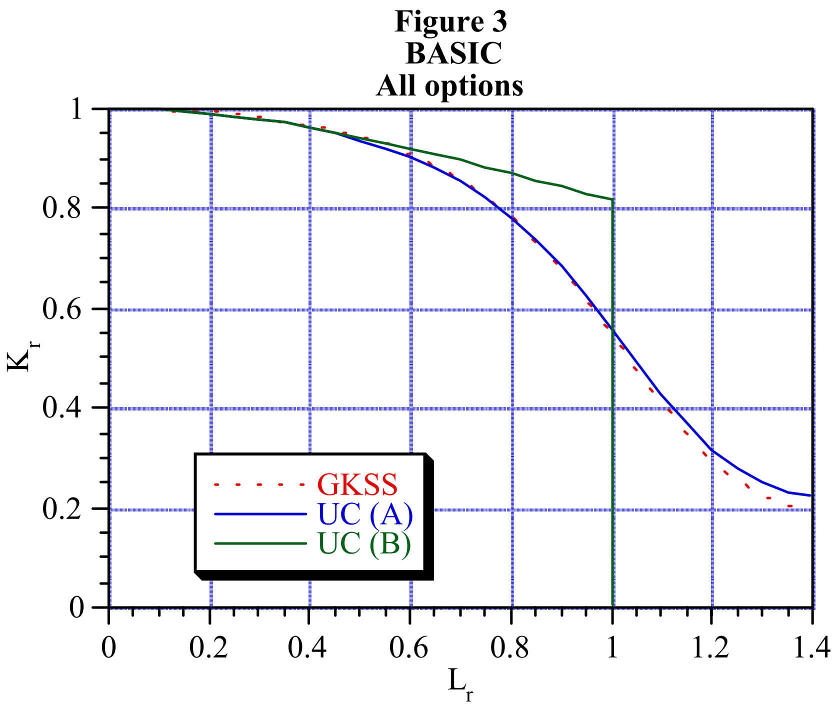


Figure 3
BASIC
All options
1
0.8
0.6
0.4
0.2
0
K
r
0
0.2
0.4
0.6
0.8
1
1.2
1.4
L
r
GKSS
UC (A)
UC (B)

Figure 4
ASSUMED N
Continuously hardening material

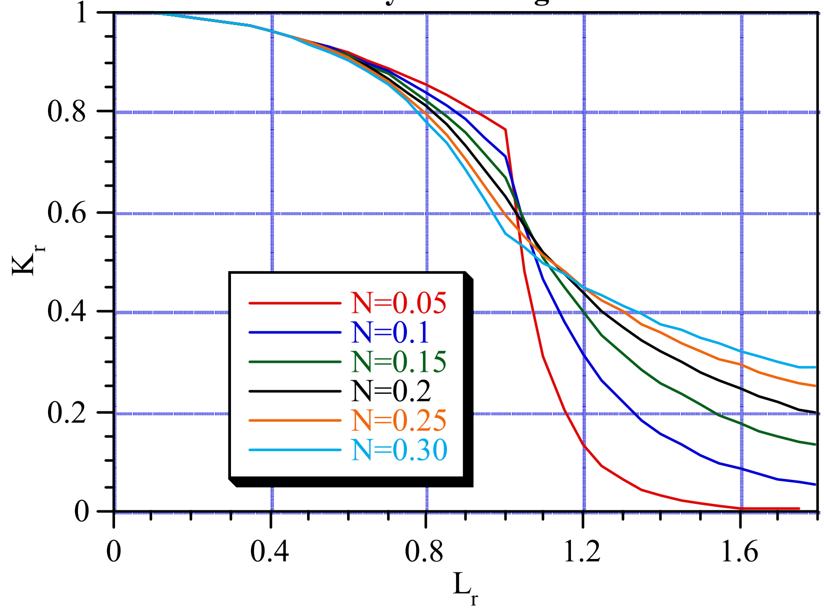


Figure 5
ASSUMED N
Material with yield plateau. Comparison

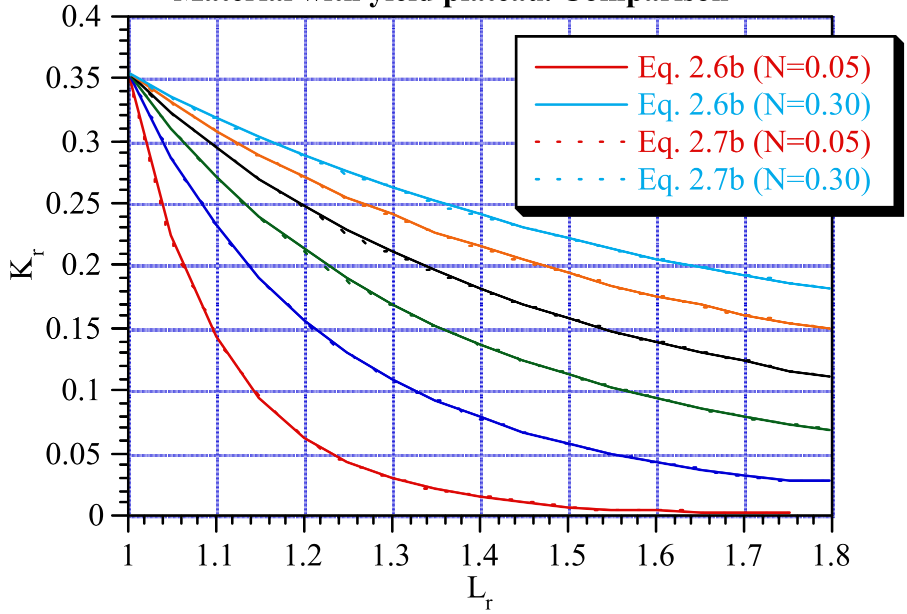


Figure 6
ASSUMED N
Material with yield plateau. Comparison

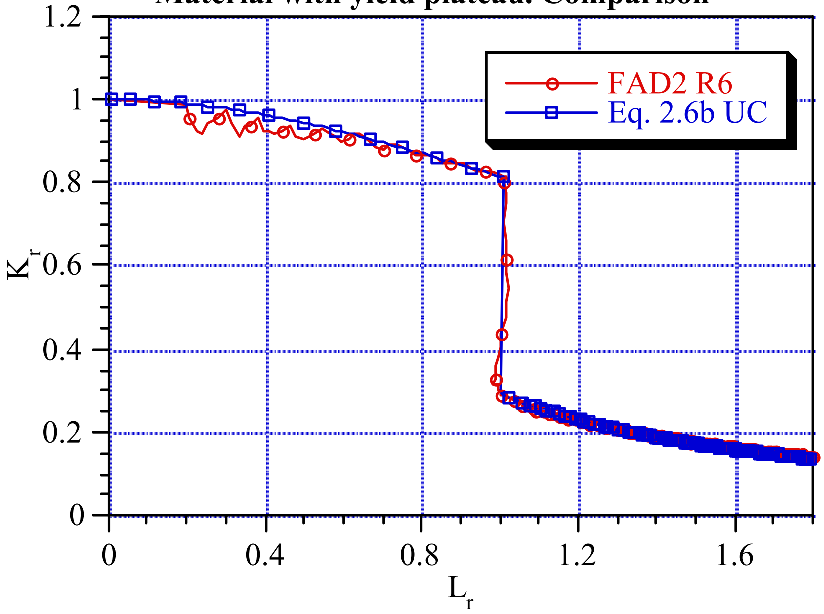


Figure 7
Stress-strain curve
Ferritic steel