The μ Factor

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1. INTRODUCTION TO SINTAP OPTION 2 "ASSUMED N"

The following points can be highlighted that summarise SINTAP Option 2 "Assumed N".

- Two types of materials (continuous and discontinuous).
- Two regions ($L_r \leq 1$ and $L_r > 1$).
- Today, only discontinuous materials in the $L_r \leq 1$ region has been solved and agreed within SINTAP.
- Present work: Strain hardening exponent as main parameter for both regions.
- Problem: Confusion and mess with definition, use and measurement of N. Also, different treatment of N needed to assure conservativeness ($L_r \leq 1$ and $L_r > 1$).

2. BACKGROUND

Lot of work has already been done about the strain hardening exponent in order to consider it as the parameter which can define the Failure Assessment Lines for the "Assumed N" option both in the elastic ($L_r \leq 1$) and in the plastic ($L_r > 1$) regions.

Also, important discussions have appeared related to this coefficient:

- should it be calculated from engineering or true values?
- should it be evaluated from a statitiscal fit or from two certain points?
- in the case of a mathematical function which is the origin of such fitting equation?

GKSS has already performed a deep study on this for the whole dominion. Nevertheless, we think that, from the starting point (Amsterdam meeting, December 2nd 1997), a mistake has been done for $L_r \leq 1$.

We are trying to evaluate the elastic regime from a hardening exponent or from the Y/T ratio. How can be study the elastic region of a material by means of the plastic values? It seems to be a contradiction. This confusion can be seen in Figure 1.

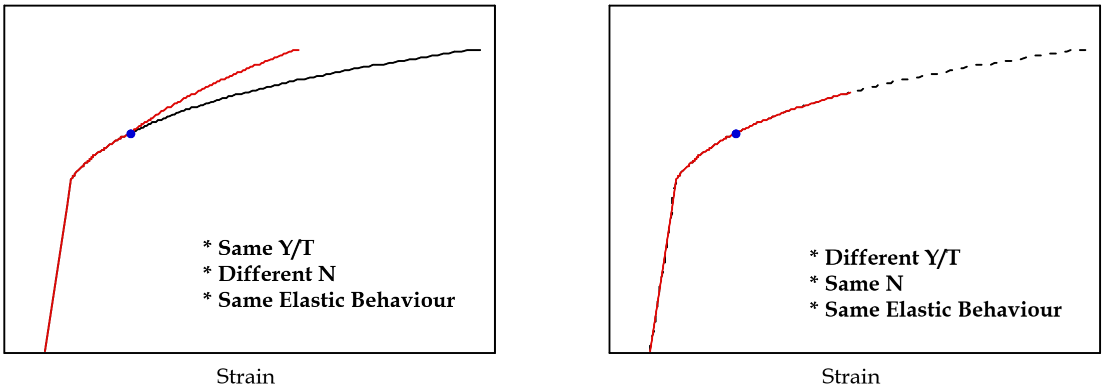

Figure 1

Therefore, we have to find out what can characterise the elastic regime easily.

It has been considered that a material behaves elastically up to σ_y but from an unknown point it leaves the linear-elastic behaviour. Thus, the stress-strain curve is not perfectly straight until the yield stress.

Referring to the Failure Assessment Line, this *surrender* to the perfectly linear-elastic behaviour is translated into a *punishment* of the function defining such behaviour:

$$K_r = \left[1 + \frac{L_r^2}{2}\right]^{-1/2} \tag{1}$$

Up till now, this *punishment* has been done through the factor:

$$\left[0.3 + 0.7\exp(-\mu N L_r^6)\right] \tag{2}$$

where a correlation of both μ and N with Y/T ratio has been tried to be found.

3. HYPOTHESIS

- The mathematical function obtained by multiplying both (1) and (2) is considered as the Failure Assessment Line for SINTAP Option 2.

- This equation should be fitted as well as possible to SINTAP Option 3 (R6 Option 2, Ainsworth's expression).

$$K_r = \left[\frac{E\varepsilon_{ref}}{L_r\sigma_y} + \frac{L_r^3\sigma_y}{2E\varepsilon_{ref}}\right]^{-1/2} \tag{3}$$

- N or Y/T cannot represent the elastic regime, so the term μN in Equation 2 is replaced by a single parameter μ that will be related to the characteristic values representing the behaviour up to σ_y.

- If a stress-strain curve is analysed, we can realise that the elastic region can be characterised by ε_y (strain at yield stress) and therefore by means of the ratio σ_y/E as $\varepsilon_y = 0.002 + \sigma_y/E$.

- So, a new working chance appears, that is, trying to fit SINTAP Option 3 curve (evaluated point by point) through the following equation:

$$K_r = \left[1 + \frac{L_r^2}{2}\right]^{-1/2} \left[0.3 + 0.7\exp(-\mu L_r^6)\right] \tag{4}$$

where, again, the first factor represents a perfect linear-elatic behaviour and the second one, a *punishment* due to the *surrender* to such behaviour.

As a first approach Eq. 4 fits Eq. 3 statistically but also an analytical approach can be done.

- From an analytical point of view, perfect compatibility between Eq. 3 and Eq. 4 can be obtained at certain values, particularly at L_r=1.

  If we want to assure that Eq. 4 is lower or equal than Eq. 3 at L_r=1, we can perform the following evaluation:

$$\begin{gathered} (\text{Eq.4})_{L_r=1} \le (\text{Eq.3})_{L_r=1} \\ \left[\frac{3}{2}\right]^{-1/2} \left[0.3 + 0.7\exp(-\mu)\right] \le \left[\beta + \frac{1}{2\beta}\right]^{-1/2} \end{gathered} \tag{5}$$

where

$$\beta = 1 + \frac{E}{\sigma_y} 0.002 \tag{6}$$

(Note: This parameter was anticipated in the Amsterdam meeting December 2nd 1997)

- Therefore, two types of calculations have been done for twelve materials:

  * **A statistical least square fit where $\mu^{statistical}$ is obtained.**

  * **An analytical ($\mu^{analytical}$) value that makes different options compatible at L_r=1.**

4. RESULTS

As mentioned above, twelve materials have been analysed. The stress-strain curves were known and hence the corresponding SINTAP Option 3 Failure Assessment Lines were obtained. After that, the different μ values were calculated. The different Failure Assessment Lines are presented in the following pages.

The studied materials are:

- Microalloyed Steel E690 (1).
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- Microalloyed Steel E500.
- Quenched Steel 4135B.
- Normalised Steel 4135A (1).
- Normalised Steel 4135A (2).
- Aged Stainless Steel.
- Stainless Weld Steel.
- Austenitic Steel.
- Aluminium.

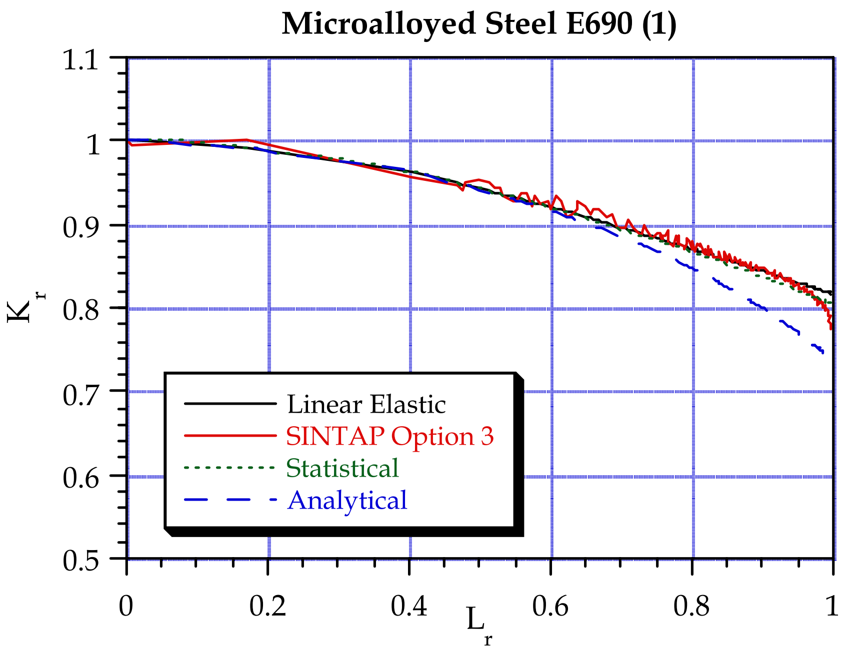

Figure 2

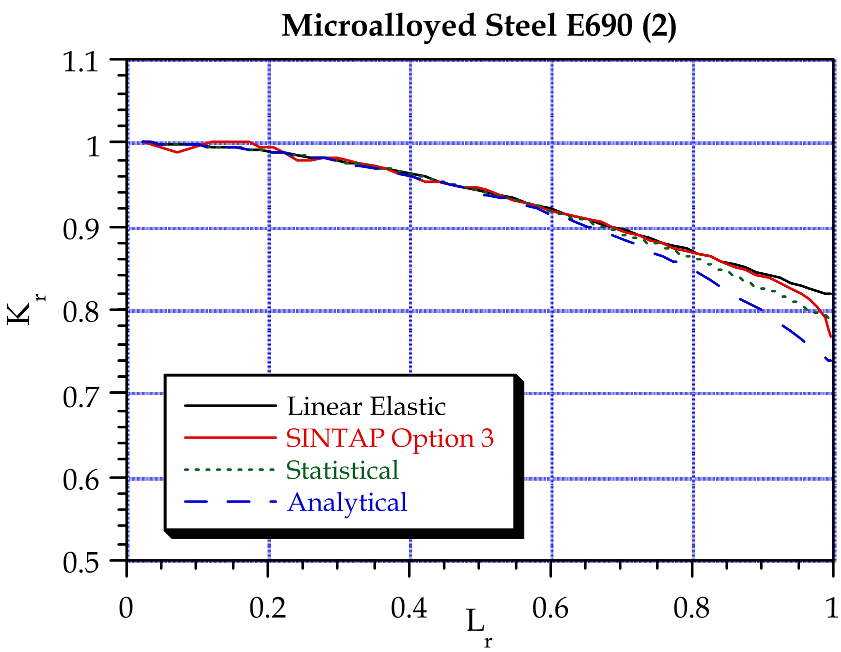

Figure 3

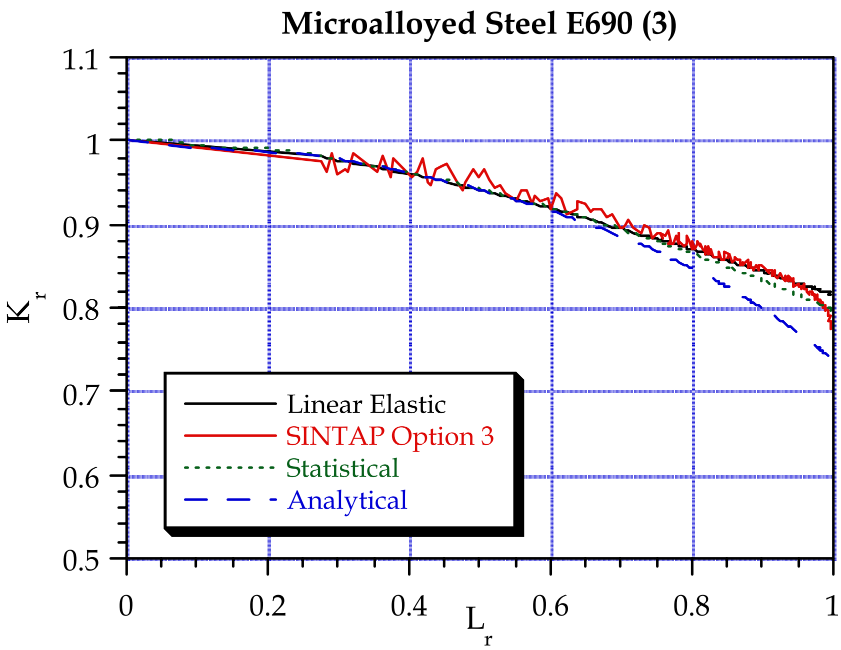

Figure 4

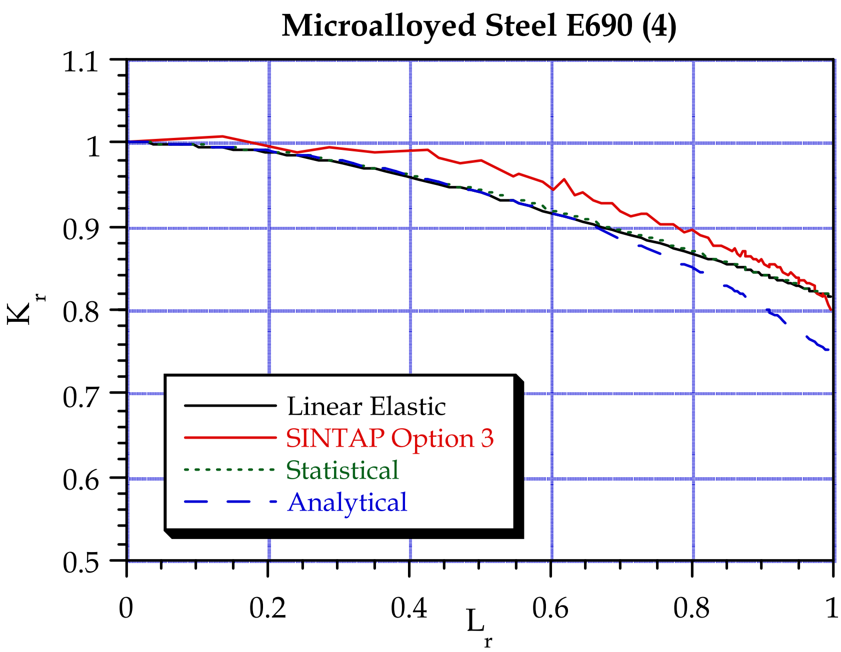

Figure 5

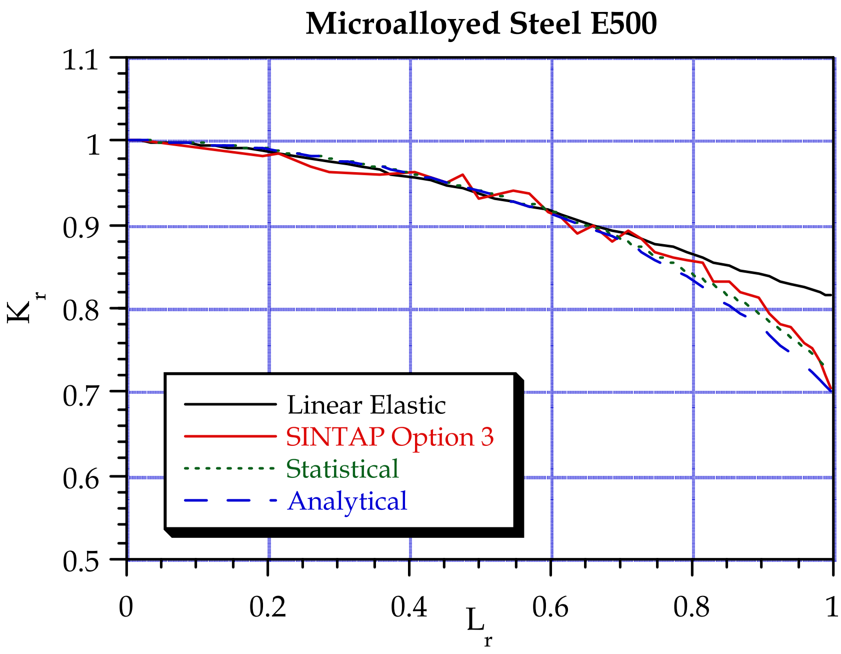

Figure 6

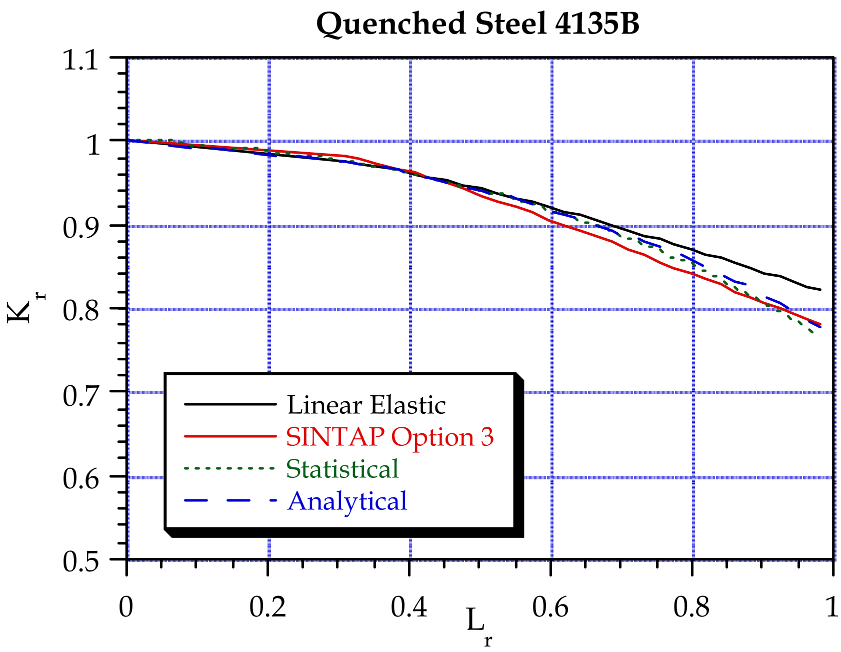

Figure 7

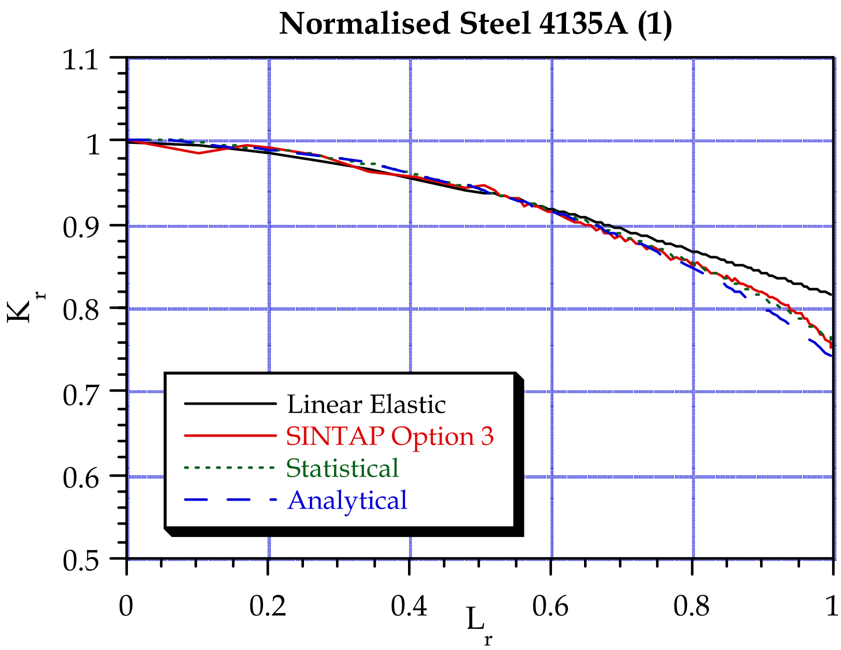

Figure 8

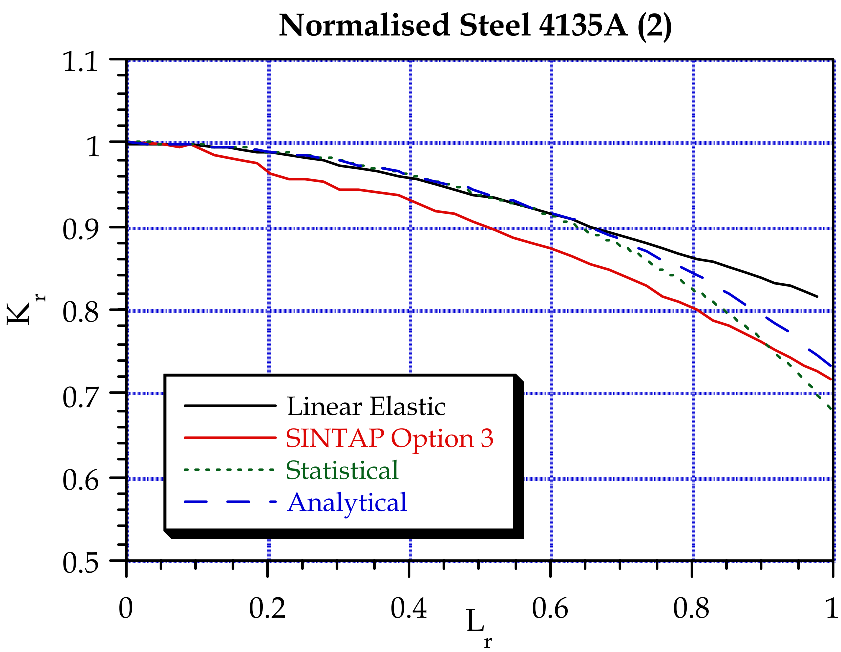

Figure 9

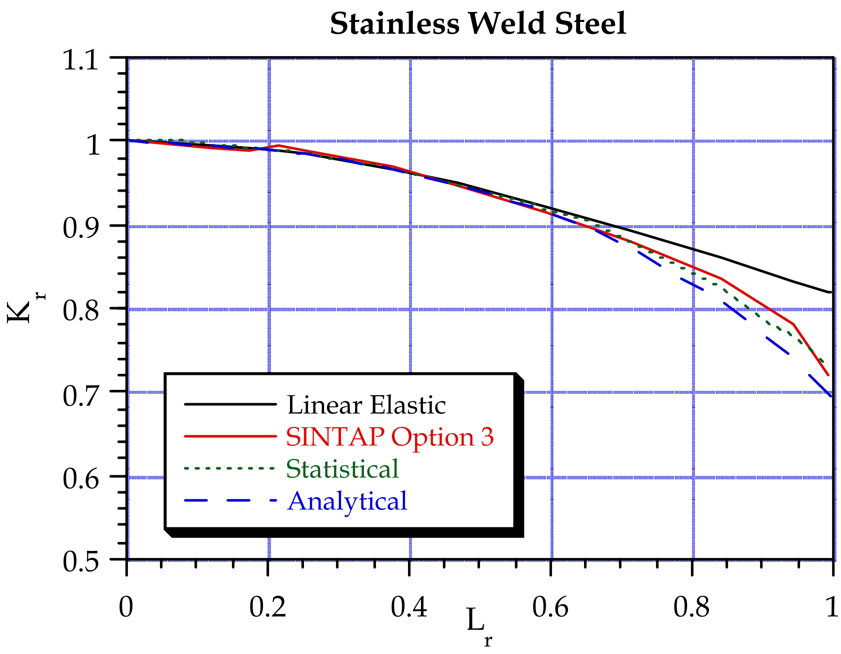

Figure 10

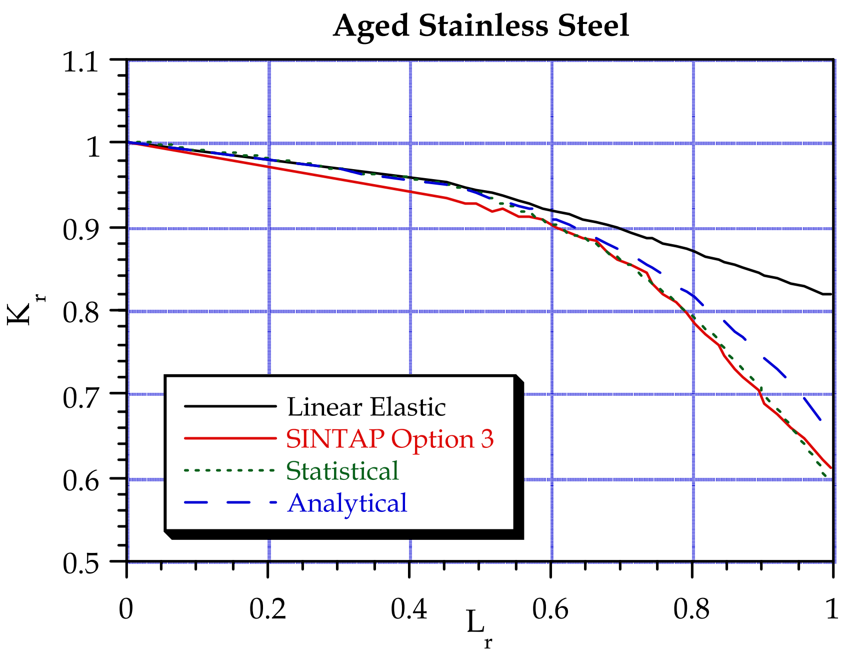

Figure 11

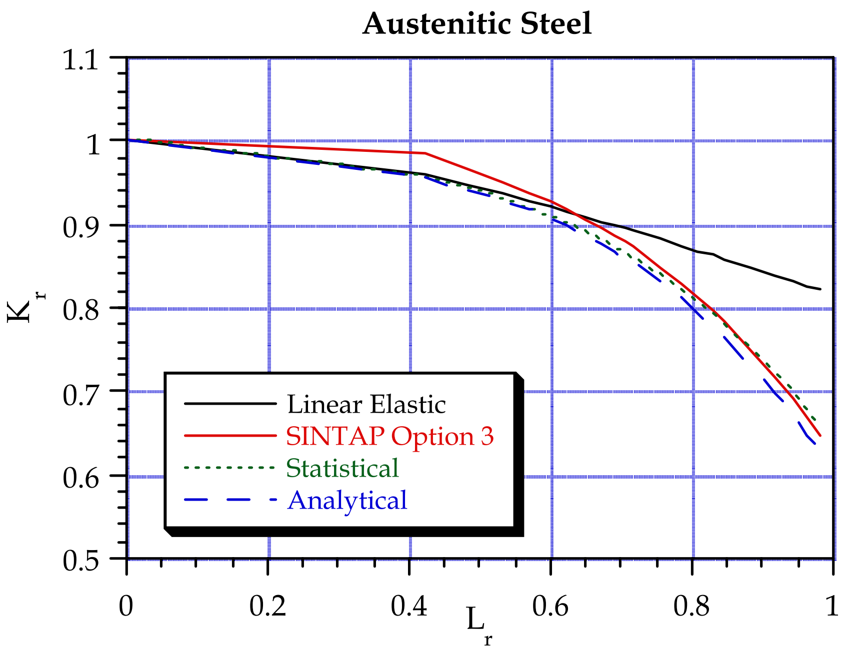

Figure 12

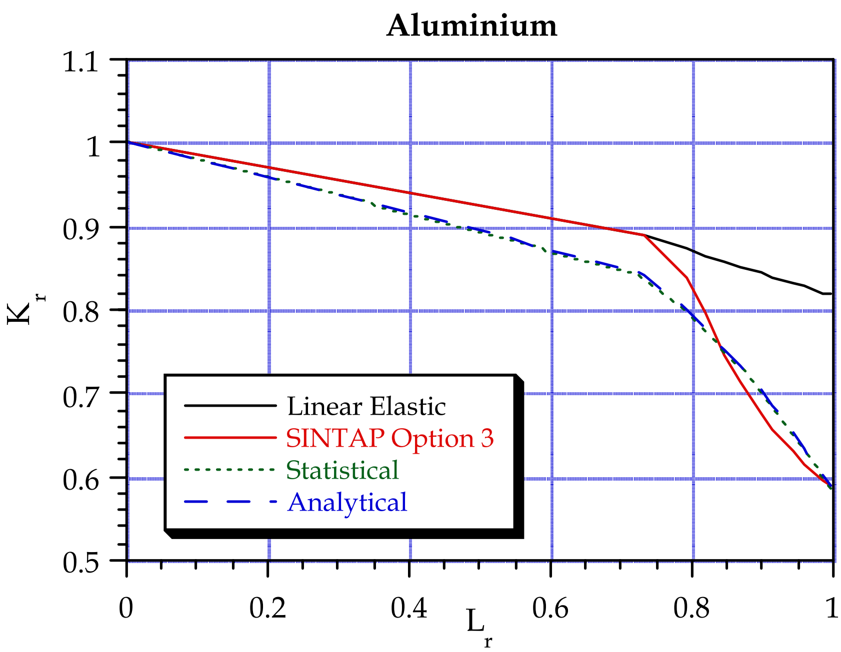

Figure 13

All these results can be summarised in the following figure where the different definitions of μ are presented versus the ratio σ_y/E.

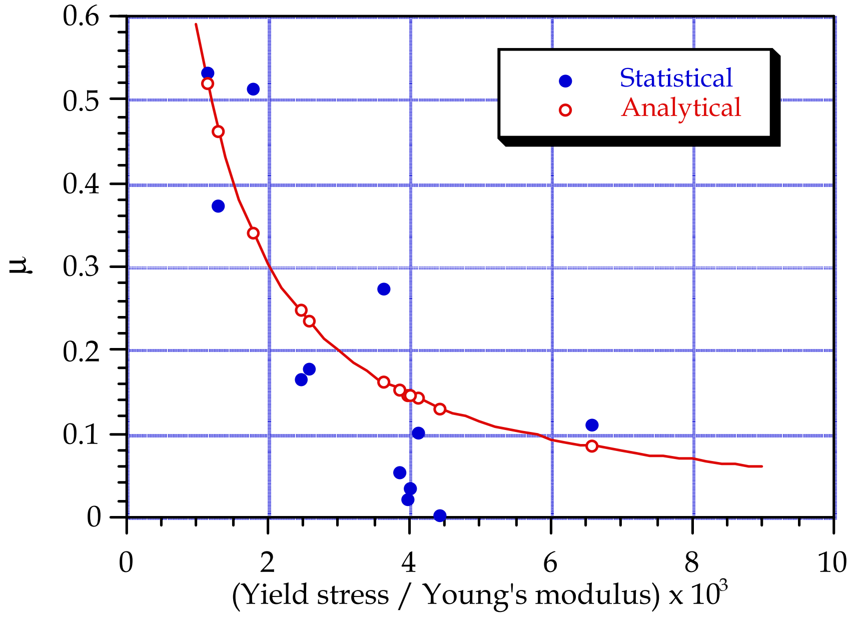

Figure 14

From Figure 14, a safety definition of μ can be done. This has been represented in Figure 15 as a first estimation.

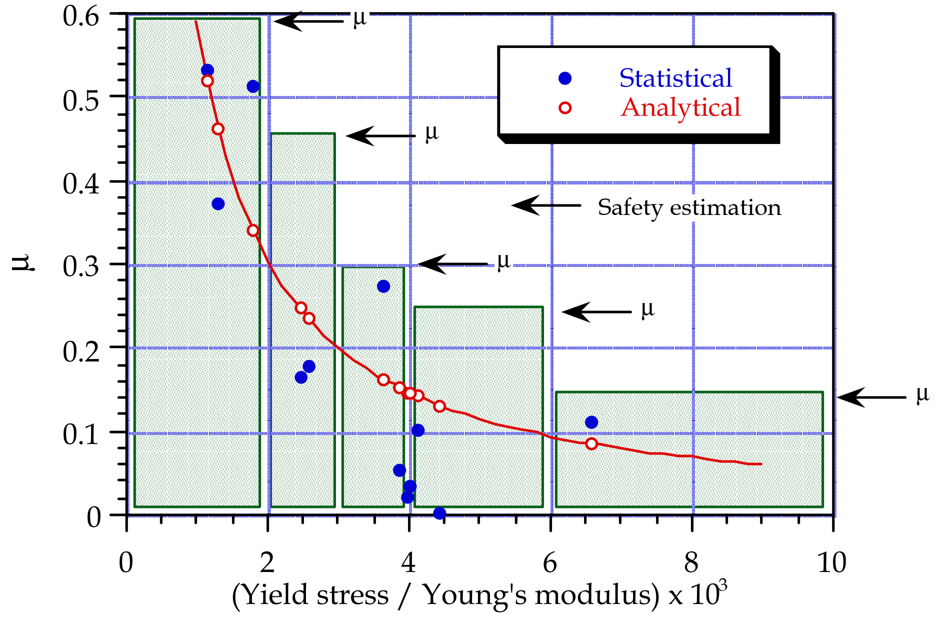

Figure 15

After Figure 15, the safety estimated values of μ have been tabulated as a preliminary suggestion.

$(\sigma_y/E) \times 10^3$	μ
0-2	0.60
2-3	0.45
3-4	0.30
4-6	0.225
>6	0.15

5. CONCLUSIONS

- The Failure Assessment Line for $L_r \leq 1$ for a continuous material can be easily calculated by means of, uniquely, the yield stress and the Young's modulus.

- There is no need of using parameters that are obtained above the yield stress. So, the elastic region is explained through elastic values.

- There is no contradiction with GKSS and other studies for $L_r>1$ since there is full compatibility with the different formulae presented by them (they depend on the K_r value for $L_r=1$, independently of the concrete value).

- Further refinements can be done. So, the study of as many stress-strain curves as possible would be needed and proper.

- Also, the coefficients 0.3 and 0.7 of the *punisher* curve can be studied deeper. Perhaps, they could be changed if a benefit can be demonstrated to exist.