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FRACTURE TOUGHNESS CONSIDERATIONS

1. BACKGROUND

The desirability of having appropriate accurate toughness data cannot be emphasised too strongly, and it is vital that toughness data be obtained from properly documented welding procedure test samples at the time of construction and extracted from all the critical weld regions including the heat-affected zones and the weld metal.

One of the key-inputs for any structural integrity assessment is the fracture toughness. In an ideal situation, appropriate fracture toughness data for use in structural integrity assessments are generated through the use of suitable fracture mechanics based toughness tests. In reality, however, actual fracture toughness data may not exist at all, or cannot be easily obtained due to lack of material or the impracticability of removing material from the actual structure. In these cases, it is necessary to base the analysis on a reliable correlation between Charpy impact energy and fracture toughness for the particular case being assessed.

Whenever possible, material's fracture toughness should be determined by an appropriate fracture-mechanics-based toughness tests using a standardised testing technique (e.g. ASTM E 1921, ESIS-P2, BS 7448 etc.) and standard geometry specimens, Section 3.5. In the case of welded structures, the tests should be carried out on samples welded with welding procedures, base materials and consumables as used for the service application and should take into account of restraint during welding and of PWHT if applicable. The welding procedures should be appropriately documented as welding procedure records according to e.g. EN 288-3 Annex A.

The test weldments from which the samples will be taken, should be subjected to appropriate mechanical and NDT testing & evaluation according to e.g. EN 288-3 and EN 25817, to ensure that the overall mechanical properties (e.g. hardness, tensile strength, impact toughness, bending angle) and weld quality level (Class A-D) have been achieved as required, with the welding procedure applied. This guarantees the compatibility between the obtained Welding Procedure Specification (WPS) and the thereby derived welding guidelines intended for use in actual welding fabrication, which further ensures that the fracture mechanics test samples will, in this respect, be representative of the actual structure/component to be assessed.

2. INTRODUCTION

Even when direct fracture mechanics based toughness data are available, the existing data can appear in various forms, as the material's fracture toughness used in a fracture mechanics analysis can be based either upon K_{IC} , J or CTOD (δ). Testing standards cannot give any recommendations for the application of fracture toughness data for structural integrity assessment, since they are based on ensuring the correct test performance and the quality of the data rather than treatment of data. A number of methods and models are available today for the treatment of fracture toughness data, especially in the ductile-to-brittle transition regime, but very few comparative studies have been conducted to validate the results.

Treatment of fracture toughness data, therefore, varies depending on the type of the data (K_{IC} , J, CTOD (δ)) that, in each case, are available for the fracture mechanics analysis. This complicates structural integrity assessment and makes it difficult to apply any single, unified procedure.

Instead of applying various equations and routes for different types of toughness data, an approach is presented here, in which one material specific K_{mat} value, together with its probability distribution $P\{K_{mat}\}$ is defined, irrespective of the type of the original toughness data available. Using the approach, a fracture toughness estimation methodology has been developed and formulated to the '*SINTAP Fracture Toughness Estimation Procedure*'.

The fracture toughness estimation methodology within 'SINTAP' has been developed for the unified treatment of various forms of toughness data for use in structural integrity assessments. The methodology, written in the form of a procedure, defines one material-specific K_{mat} value, and the fracture toughness data for the procedure are thereby based on K methods (including conversions from J and CTOD methods, Section 3.3). All other toughness data types are therefore transferred into K_{mat} which hence represents a unique parameter and its distribution describing material's fracture toughness for the assessment. The various treatments needed for the fracture toughness analysis, including e.g. specimen size adjustment are then applied directly to the K_{mat} data.

The methodology can be as easily applied to Charpy data, as to actual fracture toughness data and is suitable for treatment of data at both single and different temperatures. The evaluation procedure enables a reliable fracture toughness estimate to be obtained for various forms of data sets containing results from both homogeneous and inhomogeneous material. Thus, it can be applied not only for base materials but, in the case of welded joints, for weld metals and heat-affected zones. The Procedure allows fracture toughness assessment with quantified probability and confidence levels. For assessment against brittle fracture, the Procedure is based upon the maximum likelihood concept (MML) that uses a 'Master Curve' method to describe the temperature dependence of fracture toughness. Irrespective of the type of the original data, one material-specific K_{mat} value representing a *conservative estimate of the mean fracture toughness* is obtained together with its probability distribution $P\{K_{mat}\}$. This information can be subsequently applied to structural integrity assessments.

Section 3 outlines the general principles of the treatment of toughness data, whereas Section 4 introduces the scientific background of the statistical analysis of fracture toughness data. In Section 5, the mathematical formulation of the SINTAP Fracture Toughness Estimation Procedure comprising three steps is explained and guidance is given on the selection of a step that is most appropriate to the particular type of data available for material's fracture toughness estimation. Section 6 gives additional practice guidance for the correct application of the Procedure.

Section 7 discusses separately the treatment of data for ferritic steels on the upper shelf of fracture toughness, for materials which do not exhibit brittle cleavage fracture, or in the case that only ductile fracture data is available. This section is meant to be guidance, only.

3. PRINCIPLES OF THE TREATMENT OF TOUGHNESS DATA

The principles of the treatment of toughness data are discussed, in connection with basic definitions and conversions needed for the application of the procedure. The aspects related to tearing and constraint are discussed in the extent necessary for the material characterisation purposes.

3.1 General Characteristics of Data Treatment

Apart from the type of fracture toughness data available, the philosophy of the *design* of a structure itself can be based either on "*design against brittle fracture*" or "*design against ductile fracture*". In the former case, but having only *ductile fracture data* (J_C , J, J-R and $K_{C(ductile)}$) available, the treatment follows the Step 3 of the Procedure, but using the minimum initiation value *treated* as a brittle cleavage fracture event according to Section 5.

For ferritic steels on the upper shelf of fracture toughness, or for materials which do not exhibit brittle cleavage fracture, design against brittle fracture is not necessary. In this case, treatment of *ductile fracture data* should follow a separate approach according to SINTAP Procedure Section K and Appendix 2 (and discussed here in Section 7). Due to insufficient knowledge of the extent of to which all the factors (e.g. constraint, mismatch, scatter, definition of "initiation", testing, etc.) influencing the ductile fracture

behaviour should actually be taken into account in the prediction, the approach is suggested as guidance only.

In the case of *CTOD data* in the form of δ or J, the treatment is conducted using relevant K-CTOD-J conversions, as explained in the SINTAP Procedure Section K and presented here in Section 3.3. With respect to the parameter δ , the procedure is based on the standard CTOD (according to BS 5762) and other non-standardised parameters, like δ_5 , are given only as a reference.

In the case of *brittle K_{IC} or K_{JC} data*, the treatment continues according to the present Procedure in Sections 4 and 5 which is based upon the maximum likelihood concept (MML) that uses a 'Master Curve prediction method to describe the temperature dependence of fracture toughness. The Master Curve method makes the following assumptions: (i) specimen size adjustment, (ii) distribution of scatter and (iii) minimum fracture toughness (K_{min}) and temperature dependence. Being equally suitable to not only base material but also welded joints, the treatment also includes the homogeneity check.

In the case that *Charpy data* is all that is available for the assessment, the treatment is carried out according to the SINTAP Default Procedure Appendix 1. The default level uses Charpy correlations for the cases where (i) less than 3 fracture toughness tests are available, or (ii) only Charpy data is available. The SINTAP Fracture Toughness Estimation Procedure (in SINTAP Procedure Section K) defines the different operational levels. In the case of *brittle fracture*, Level 1 for the number of tests between 3 and 9 uses the MML concept with Steps 1, 2 and 3, together with an additional safety factor accounting for the small number of tests. Level 2 for more than 9 test results allows the use of either Step 1 or 2 estimate in the case of 20 % (or higher) failure probability, with Step 3 used only as information of possible material inhomogeneity. Provided that a failure probability of less than 20 % is required, Level 2 uses the complete procedure with Step 1, 2 and 3 estimates, as described in Section 5. In the case of *ductile fracture*, the default level uses the minimum of three J_R -curves, Level 1 and Level 2 being equivalent, see Section 7.

The Procedure is initially structured in a way that the less sufficient/accurate the original data to be converted to K_{mat} , the more it will be penalised in the probabilistic fracture mechanics assessment. While this ensures that even the estimate derived from 'lowest quality data' is always 'safe', it also guarantees that the more sufficient/accurate the original data, the less becomes the need for additional procedure-induced conservatism. This way, the procedure rewards the user that has the most accurate data but, on the other hand, permits the maximum benefit to be gained from any given data set by relating the penalty to the quality of the original data. Any additional data improving the accuracy of a previously existing data set can also be readily utilised in terms of reduced conservatism. The procedure not only enables the quantification of probability and confidence levels of the K_{mat} estimate, but also guarantees the avoidance of multiple safety margins that could lead to unnecessary conservatism.

Because of the maximum likelihood concept (MML) chosen as a basis of the fracture toughness evaluation, the methodology allows the use of data sets consisting of both results ending in failure and those ending in non-failure. Therefore the whole data set that is available in each case can always be fully utilised in the analysis, regardless of whether the results are ductile or brittle. The procedure can be applied as easily to either fracture toughness data at a single temperature or to the data at different temperatures.

3.2 Definition of K_{mat}

The material toughness expressed in terms of the stress intensity factor is termed ' K_{mat} ' in this document. Generally, the fracture toughness can be based upon K_{IC} , J, or CTOD. However, regardless of parameter it is preferable to express the fracture toughness in terms of its equivalent K-value, denoted here as K_{mat} . When linear-elastic plane strain fracture toughness K_{IC} values are used, K_{mat} should be taken as K_{IC} and should be established using relevant standardised methods, such as BS 7448.

If valid K_{IC} data are not obtained, equivalent K_{mat} values from J-based tests or CTOD tests shall be used. In the case of data in the form of δ or J, the treatment should be conducted using relevant K-CTOD-J conversions in accordance with SINTAP Procedure Section K (see Section 3.3).

Tests should be carried out on standard geometry specimens wherever possible, but where assessments involve certain types of flaw and flaw orientation (i.e. shallow surface flaws, or HAZ regions), alternative specimen geometry may be appropriate. The present Annex 5 provides information on optimum treatment of fracture toughness data.

3.3 Relationship between K, J and CTOD

The representative toughness parameter for use in this procedure is based on K_{mat} . Since K, J and CTOD values can all be generated in a fracture toughness test, in some instances it may therefore be necessary to convert between these parameters.

The same function $f(L_r)$ is used for estimation of crack driving force by J and CTOD (δ) methods and for the Failure Assessment Diagram approaches. Therefore, the same solution will be given by J_{mat} and δ_{mat} approaches provided a single value of material toughness, K_{mat} is used which is related to critical values of J and δ (J_{mat} and δ_{mat}) by a consistent set of equations. The equations are in compliance with SINTAP Procedure Section K.

J_{mat} is related to K_{mat} by:

$$K_{mat} = \sqrt{JE / (1 - \nu^2)}$$

δ_{mat} is related to K_{mat} by:

$$K_{mat} = \left[\frac{\lambda s_x d_{mat} E}{(1 - \nu^2)} \right]^{0.5}$$

where for steels other than duplex stainless, λ should be taken as 1.5 for $\sigma_x = \sigma_y$ and 1.3 for $\sigma_x = \sigma_f$. The corresponding values for duplex and super-duplex stainless steels are 2.3 and 1.8 : flow stress σ_f is the average of the yield stress and ultimate tensile strength. The lowest of the two values calculated in accordance with the above expressions, should be used as the K_{mat} for subsequent analysis.

It is generally recognised that $\lambda = 1$ and that the term $(1 - \nu^2)$ be omitted for plane stress, while $\lambda = 2$ and the $(1 - \nu^2)$ term retained for plane strain (when using σ_y). However, it should be recognised that the value of λ depends more generally on constraint, strain hardening and on the precise manner in which CTOD is measured.

In the Procedure as it currently stands only the simplified use of CTOD as currently represented by the above equations with λ is given. This does not, however, preclude the use of more detailed CTOD methods such as those within the ETM procedures.

3.4 Tearing Analysis

For tearing analysis, reference is made to SINTAP Procedure Appendix 2.

3.5 Constraint Effect on Toughness

Constraint is discussed here in terms of specimen's size and geometry related aspects and only to the extent which is necessary for the material's toughness characterisation purposes. For the inclusion of constraint analysis in structural integrity assessments, guidance is provided in SINTAP Procedure Appendix 4.

3.5.1 Specimen Thickness Effect

Standardised CTOD testing usually prescribes the use of full thickness specimens, regardless of the actual plate thickness. Because the deformation characteristics and flaw geometry of a real structure may

have very little in common with those of the CTOD specimen, there is practically no guarantee that the CTOD values are really descriptive of the structure. J-based testing standards, like ASTM E 1921 set specific criteria for the specimen size in order to ensure descriptive results.

The specimen should be sufficiently large to allow for the crack driving force level that is comparable with that of the particular structure/component whose structural integrity is being assessed. This, however, does not automatically mean full-thickness specimens, provided that already smaller specimens allow for the structural crack-driving force level to be obtained.

The user should, however, recognise that the SINTAP Procedure is of such the nature where the use of too small specimens penalises the toughness estimate. The inaccuracy towards a conservative direction arises from the size-adjustment of the data to correspond the specimen thickness of 25 mm.

In the material's ductile-to-brittle transition region where cleavage fracture becomes a dominant mechanism, descriptive results are guaranteed by inclusion of the specimen's size criterion that ensures the measuring capacity which is great enough to avoid large-scale yielding conditions. The specimen's measuring capacity limit depends on the specimen geometry and material properties and can be calculated according to relevant test standards, such as ASTM E 1921-97 according to SINTAP Procedure Section K and presented here in Section 5. Inclusion of the specimen's size criterion, guaranteeing descriptive results, combined with statistical analysis enables small-scale fracture toughness data to be reliably used for the failure assessment of large structures/components, regardless whether the specimen's failure mode is ductile or brittle.

3.5.2 Geometry Effect on Toughness

Standardised testing methods (e.g. BS 7448) used to derive fracture toughness data are necessarily conservative and represent high constraint applications. Guidance is provided in SINTAP Procedure Appendix 4 for the inclusion of constraint analysis in structural integrity assessments.

For toughness tests on welded joints, it is essential that the measurement takes account of differences in properties between the weld metal, the heat-affected zone and the parent material and also of the deformation pattern which actually occurs.

4. STATISTICAL ANALYSIS OF FRACTURE TOUGHNESS DATA

The treatment of fracture toughness data (K_{IC} , K_{JIC} , J_{IC} , J_I , $J-R$, $K_{IC(ductile)}$) can be classified as either design against brittle fracture or design against ductile fracture (with either brittle or ductile fracture data available). The methodology is described in detail in the present document.

For ferritic materials operating in their transition regime or close to the lower shelf, the assessment must be based on *design against brittle fracture*. The procedure for the analysis of data is based upon the maximum likelihood concept (MML) that uses a 'Master Curve' method to describe the temperature dependence of fracture toughness. The procedure makes the following assumptions: (i) specimen size adjustment, (ii) distribution of scatter and (iii) minimum toughness and temperature dependence. Being equally applicable to welded joints, (iv) a data homogeneity check is included. As a result, a conservative estimate of the mean fracture toughness (and its distribution) is obtained. The method is in compliance with the ASTM E 1921-97.

For materials with their operating temperature in the upper-shelf regime, or materials which do not exhibit brittle cleavage fracture, design against brittle fracture is not necessary. Therefore, the analysis of data follows a separate approach for *design against ductile fracture* and is presented in Section 7. Due to insufficient knowledge of the extent to which e.g. constraint, mismatch, scatter, definition of 'initiation', testing etc. influencing the fracture behaviour should be considered, this approach is not meant as a procedure, but is suggested as guidance only.

4.1 Scatter and size effect of fracture toughness at cleavage instability

The fracture toughness to be used in a fracture mechanics analysis can be based upon K_{IC} , J , or CTOD. Regardless of parameter it is preferable to express the fracture toughness in terms of its equivalent K value, denoted here as K_{IC} .

The Procedure assumes the scatter to follow the statistical brittle fracture model which uses a Weibull type distribution function to describe *scatter in fracture toughness* as:

$$P[K_{IC} \leq K_I] = 1 - \exp\left(-\left[\frac{K_I - K_{min}}{K_0 - K_{min}}\right]^4\right)$$

where $P[K_{IC} \leq K_I]$ - i.e. P_f - is the cumulative failure probability at a K_I level, K_I is the stress intensity factor level, K_{min} is the lower bound to the fracture toughness and K_0 is a temperature and specimen thickness dependent normalisation fracture toughness which corresponds to a 63.2 % cumulative failure probability (and is approximately $1.1 \bar{K}_{IC}$ where \bar{K}_{IC} is mean fracture toughness).

The methodology predicts a *statistical size effect of fracture toughness* test specimens of the form:

$$K_{B2} = (K_{B1} - K_{min})(B_1 / B_2)^{1/4} + K_{min}$$

where B_1 and B_2 correspond to respective specimen thickness (length of crack front).

Other statistical brittle fracture models yield very similar equations, the main difference being essentially in the treatment of K_{min} . Although " K_{min} " itself can be regarded as "theoretical" in nature, for structural steels a fixed, experimental value of $K_{min} = 20 \text{ MPa}\sqrt{\text{m}}$ can be used.

The model here is based upon the assumption that brittle fracture is primarily initiation controlled, even though it contains a conditional crack propagation criterion, which among others results in the lower bound fracture toughness K_{min} . Close to the lower shelf of fracture toughness ($K_{IC} < 50 \text{ MPa}\sqrt{\text{m}}$) the equations are expected to be inaccurate because the initiation criterion is no longer dominant, and the macroscopic fracture is propagation controlled. In this case there is no statistical size effect and the toughness distribution differs slightly from the Weibull type distribution function.

In the ductile to brittle transition region the equations presented here should be valid as long as loss of constraint and/or ductile tearing do not play a significant role.

4.2 Temperature dependence of fracture toughness

The 'Master Curve' is used in the ASTM standard (E 1921-97) for fracture toughness testing in the ductile to brittle transition region. It describes the temperature dependence of fracture toughness K_0 , which for ferritic structural steels is:

$$K_0 = 31 + 77 \cdot \exp(0.019 \cdot [T - T_0])$$

where T_0 ($^{\circ}\text{C}$) is the transition temperature where the mean fracture toughness, corresponding to a 25 mm thick specimen, is $100 \text{ MPa}\sqrt{\text{m}}$ and $K_0(T_0)$ which is a normalisation fracture toughness at 63.2 % cumulative failure probability, is $108 \text{ MPa}\sqrt{\text{m}}$.

The expression gives an *approximate* temperature dependence of the fracture toughness for ferritic structural steels and it is comparatively well verified. The effect that possible outlier or invalid fracture toughness values may have upon the transition temperature T_0 decreases if the temperature dependence is fixed.

4.3 Homogeneity Check

Since the SINTAP Procedure is equally applicable to welded joints it contains a data homogeneity check. Generally, in the case of 'homogeneous' data it is sufficient to consider only the toughness of the matrix microstructure and the estimate can be based on the mean value of the data. In the case of 'inhomogeneous' data, where the 'brittle microstructure' is substantially more brittle (e.g. 3 times) than the 'matrix microstructure', the fracture behaviour will be dominated by the brittle microstructure alone and, consequently, the estimate must be based on the minimum value of the data.

5. FRACTURE TOUGHNESS ESTIMATION: THE SINTAP PROCEDURE

This Section describes the details of the three steps ('stages' according to SINTAP Procedure Section K) of the Fracture Toughness Estimation Procedure, with the associated mathematical equations. The nomenclature of the mathematical equations the Procedure uses are given in the following:

B	= specimen thickness
b_0	= size of the uncracked ligament (= $W - a_0$)
σ_y	= material's yield strength
P_f	= cumulative failure probability
K_{CENS}	= censoring value for individual fracture toughness
K_{limit}	= specimen measuring capacity as defined in testing standards
K_{MAT}	= individual fracture toughness
$K_{\text{MAT}25}$	= size adjusted K_{MAT}
\bar{K}_{MAT}	= median K_{MAT}
K_0	= 63.2 % failure probability K_{MAT}
$P\{K_{\text{MAT}}\}$	= probability distribution corresponding to the median K_{MAT} estimate
N	= total number of tests (including censored specimens)
r	= number of toughness values (specimens) corresponding to brittle failure
T_0	= $100 \text{ MPa}\sqrt{\text{m}}$ \bar{K}_{MAT} transition temperature
T	= operating temperature
δ	= censoring parameter: $\delta = 1$ (brittle), $\delta = 0$ (censored)

The procedure for statistical analysis of toughness data can be applied to ferritic structural steels to indirect fracture toughness (Charpy) data, as well as to actual fracture toughness data and is suitable for treatment of data at both single and different temperatures. The data sets may contain results from both homogeneous and inhomogeneous material, making the procedure also applicable to welded joints. The procedure allows reliable fracture toughness assessment with quantified probability and confidence levels. Irrespective of the type of the original data, one material-specific K_{mat} value representing a conservative estimate of the mean fracture toughness is obtained together with its probability distribution. This information can be subsequently applied to structural integrity assessments.

The maximum likelihood concept (MML) which the present methodology uses for the toughness estimation, suits well to analysis of data sets which include both results ending in failure and non-failure. This is often the case in practice fracture mechanics testing, especially for welded joints. Depending whether the original data consist of results obtained at different temperatures or correspond to one single temperature, the parameters to estimate are either the transition temperature T_0 or the normalisation toughness K_0 , respectively. As a result of the procedure, a conservative estimate the mean fracture toughness is obtained.

The procedure progresses according to three separate steps: Step 1: Normal Maximum Likelihood Estimation, Step 2: Lower-Tail Maximum Likelihood Estimation and Step 3: Minimum Value Estimation. Depending on the characteristics of the original data available, the procedure guides the user towards the Step that gives the most appropriate toughness estimate for the fracture toughness analysis to the particular case being assessed.

Each Step sets a different validity level for that part of the data that is to be censored. The whole data set is involved in the analysis, however, a certain pre-assumption is made concerning the nature of the data being censored. For example, non-brittle results that are above the censoring validity level, are recognised to be higher than the validity level, but the toughness value corresponding to this validity level is used in further estimation.

5.1 Calculation of K_{mat} and Size Adjustment

Firstly, the available data is written in the form of K_{mat} . In the case that the original data appears in the form other than K (e.g. J, CTOD (δ), CVN etc.), the formulae according to SINTAP Procedure Section K and described here in Section 3.3 are used to convert the data in the form of K_{mat} . Otherwise, the procedure

described here should be followed, starting with size-adjustment of the original fracture toughness (K) data, which should always be made for specimens of thickness other than 25 mm.

The size adjustment is made to the KMAT data according to Eq. (1), whenever a given data set includes results from specimens having thickness other than 25 mm. The thickness adjustment for obtaining normalised, size-adjusted KMAT, denoted as KMAT₂₅, is calculated as:

$$K_{MAT_{25}} = 20MPa\sqrt{m} + (K_{MAT} - 20 MPa\sqrt{m}) \cdot \left(\frac{B}{25mm}\right)^{1/4} \quad (1)$$

where K_{MAT} is the individual (original) fracture toughness value and B is specimen thickness.

5.2 Step 1: Normal MML Estimation

In Step 1 (*Normal MML Estimation*), all the available data is used for MML estimation of K_{mat} (for data at a single temperature) or $T_0(K_{mat})$ (for data at different temperatures), with the exception of (i) all ductile results ending in non-brittle failure and (ii) those results which are affected by large-scale yielding thereby exceeding the specimen's measuring capacity limit. In the latter case, the fracture mechanical parameters no longer describe the cleavage fracture process zone correctly. Therefore, all ductile results ending in non-failure and those results which are affected by large-scale yielding are censored from a given data set.

The specimen measuring capacity limit depends on the specimen geometry and material properties and can be determined according to the relevant test standards. The ASTM standard E 1921-97 for fracture toughness testing in the transition region defines the specimen measuring capacity as: $K_{VC(limit)} = (E \times b_0 \times \sigma_{ys} / 30)^{0.5}$.

The results associated with brittle and non-brittle failure modes are designated as $\delta_i = 1$ and $\delta_i = 0$, respectively. For each censored result with the true value above the censoring validity limit, i.e. $K_{MATi} > K_{limit}$, the toughness value corresponding to this validity limit is used for further estimation of fracture toughness (i.e. $K_{MATi} = K_{limit}$) and the result is designated as being non-brittle, i.e. $\delta_i = 0$.

The procedure then has two alternative routes depending on whether test data is available (a) at a single temperature, or (b) at different temperatures.

5.2.1 Data at a single temperature

For data at a single temperature, two fracture toughness parameters: K_0 corresponding to a 63.2 % cumulative failure probability K_{MAT} and median K_{MAT} (\bar{K}_{MAT}) corresponding to the median (50 %) failure probability are calculated according to Eq. (5a) and Eq. (3), respectively, as:

$$K_0 = 20MPa\sqrt{m} + \left(\frac{\sum_{i=1}^N (K_{MAT_i} - 20MPa\sqrt{m})^4}{\sum_{i=1}^N d_i} \right)^{1/4} \quad \dots\dots\dots(5a)$$

where K_{MATi} is individual fracture toughness of a specimen (size-adjusted according to Eq. (1) whenever necessary), N is total number of tests and δ_i is censoring parameter for each individual K_{MATi} result (brittle: $\delta_i = 1$, non-brittle = censored: $\delta_i = 0$). The thereby calculated K_0 value is then used to determine \bar{K}_{MAT} corresponding to 50 % failure probability according to Eq. (3), as:

$$\bar{K}_{MAT} = 20MPa\sqrt{m} + (K_0 - 20MPa\sqrt{m}) \cdot 0.91 \quad \dots\dots\dots(3)$$

5.2.2 Data at different temperatures

In the case of having data at different temperatures, transition temperature T_0 corresponding to the 100 MPa \sqrt{m} median (50 %) K_{MAT} transition temperature is calculated iteratively from Eq. (5b) as:

$$\sum_{i=1}^n \frac{\delta_i \cdot \exp\{0.019 \cdot [T_i - T_0]\}}{11 + 77 \cdot \exp\{0.019 \cdot [T_i - T_0]\}} - \sum_{i=1}^n \frac{(K_{MAT_i} - 20 \text{ MPa}\sqrt{m})^4 \cdot \exp\{0.019 \cdot [T_i - T_0]\}}{(11 + 77 \cdot \exp\{0.019 \cdot [T_i - T_0]\})^5} = 0 \dots (5b)$$

where T_i is individual transition temperature of a specimen of toughness K_{MAT_i} , T_0 is median K_{MAT} transition temperature corresponding to 100 MPa \sqrt{m} : ($T_0(\bar{K}_{MAT})$), δ_i is censoring parameter for each individual result (brittle: $\delta_i = 1$, non-brittle = censored: $\delta_i = 0$), N is total number of tests and K_{MAT_i} is individual fracture toughness of a specimen (size-adjusted according to Eq. (1) whenever necessary).

After obtaining either $T_0(K_{MAT})$ corresponding to the 100 MPa \sqrt{m} median (50 %) transition temperature from Eq. (5b), or \bar{K}_{MAT} corresponding to the median (50 %) failure probability from Eqs (5a) and (3), the procedure then continues to the analysis according to Step 2.

5.3 Step 2: Lower Tail MML Estimation

The Step 2 procedure (*Lower-Tail MML Estimation*) is mathematically similar to Step 1 procedure in terms of equations used, the only difference being the criteria for the data to be censored.

The purpose of Step 2 is to check whether the upper tail of a given data set has a significant influence on the estimated fracture toughness \bar{K}_{MAT} . In the case that this would push the estimate according to Step 1 towards unconservative direction, Step 2 avoids this by censoring the 50 % upper tail of the data set. In other words, it is recognised that the true value of each censored result is above the censoring validity limit (i.e. $\delta_i = 0$), but the toughness value corresponding to this validity limit is used for further estimation of fracture toughness (i.e. $K_{MAT_i} = K_{CENSI}$).

Consequently, the data corresponding to a cumulative probability of 50 % or lower (i.e. the 50 % lower tail) is used for MML estimation of K_{mat} or $T_0(K_{mat})$. The purpose of this is to obtain an estimate which, besides macroscopic inhomogeneity, would also be unaffected by excessive ductile tearing or large-scale yielding. The Lower-Tail MML estimation therefore aims at obtaining a 'realistic' toughness estimate that would be descriptive of material properties only, without a risk of being influenced or violated by those results in the data set that can exhibit unrealistically high 'apparent' toughness values due to e.g. testing conditions rather than 'inherent' (microscopic) material properties.

The Step 2 proceeds as a continuous iteration process, until the 'constant' level for either K_0 or T_0 has been reached, that is, $K_0 \geq K_{0i-1}$ and $T_0 \leq T_{0i-1}$, respectively.

Firstly, the censoring value for an individual fracture toughness result, K_{CENSI} , is set either as (i) median K_{MAT} (\bar{K}_{MAT}) or is calculated as (ii) $K_{CENSI} = 30 + 70 \cdot \exp\{0.019 \cdot (T_i - T_0)\}$ corresponding to the median (50 %) failure probability level, depending on whether data at a single temperature or data at different temperatures, respectively, are going to be used for the further Step 2 analysis.

In accordance with Step 1, the results associated with brittle and non-brittle (censored) failure modes are designated as $\delta_i = 1$ and $\delta_i = 0$, respectively. For each censored individual result with the true value above the censoring value, i.e. $K_{MAT_i} > K_{CENSI}$, the toughness value corresponding to this censoring value is used for further estimation of fracture toughness (i.e. $K_{MAT_i} = K_{CENSI}$) and the result is designated as being non-brittle, i.e. $\delta_i = 0$.

The Step 2 then proceeds according to two alternative routes, depending on whether test data is available (a) at a single temperature, or (b) at different temperatures.

5.3.1 Data at a single temperature

In the case of data at a single temperature, two fracture toughness parameters: K_{δ} corresponding to a 63.2 % cumulative failure probability K_{MAT} and median K_{MAT} (\bar{K}_{MAT}) corresponding to the median (50 %) failure probability are calculated for the first iteration round 'i' according to Eq. (5a) and Eq. (3), respectively, see section 5.2.1.

Provided that the fracture toughness of an individual result given by the last iteration round is still lower than that given by the previous round, i.e. $K_{\delta} < K_{\delta-1}$, the iteration process is continued and this last obtained value, K_{δ} , is thereby set as an input value for further calculation of the next iteration round. This way, the iteration process is continued as long as fracture toughness given by the last iteration round is equal to or higher than the value given by the second last iteration round, i.e. $K_{\delta} \geq K_{\delta-1}$.

The iteration process then stops, and the two \bar{K}_{MAT} values obtained according to Eq. (5a) and Eq. (3) from Step 1 and Step 2 are taken as reference values to be compared against the \bar{K}_{MAT} estimate that will be obtained from Step 3 in the next stage.

5.3.2 Data at different temperatures

In the case of data at different temperatures, transition temperature T_{δ} corresponding to the transition temperature of an individual result for an iteration round 'i' is calculated iteratively from Eq. (5b), see section 5.2.2.

Provided that the transition temperature given by the last iteration round is still higher than that given by the previous round, i.e. $T_{\delta} > T_{\delta-1}$, the iteration process is continued and this last obtained value, T_{δ} , is thereby set as an input value for further calculation of the next iteration round. This way, the iteration is continued as long as the transition temperature of an individual result given by the last iteration round is equal to or lower than the value given by the second last iteration round, i.e. $T_{\delta} \leq T_{\delta-1}$.

The iteration process then stops, and the two $T_{\delta}(\bar{K}_{MAT})$ values corresponding to the 100 MPa \sqrt{m} median (50 %) K_{MAT} level and obtained according to Eq. (5b) from Step 1 and Step 2 are taken as reference values to be compared against the $T_{\delta}(\bar{K}_{MAT})$ estimate that will be obtained from Step 3 in the next stage.

5.4 Step 3: Minimum Value Estimation

The purpose of Step 3 (*Minimum Value Estimation*) is to check material's inhomogeneity in a given data set by assessing the significance of a single minimum fracture toughness test result. This is to avoid unconservative fracture toughness estimates which may arise if median (50 %) fracture toughness is used for a material or welded joint expressing significant microscopic inhomogeneity.

Consequently, only one toughness value corresponding to one single temperature - i.e. the minimum value in a given data set - is taken as an input value for the estimation of either median K_{MAT} , i.e. (\bar{K}_{MAT}), or $T_{\delta}(\bar{K}_{MAT})$ in the cases of data at one single temperature and at different temperatures, respectively. A criteria is then set to the allowable difference between the median (50 %) fracture toughness and the lower-bound (5 %) fracture toughness level, for both data at a single temperature and data at different temperatures. This is to assess the significance of a single minimum fracture toughness test result in a given data set. Step 3 is equivalent to bias-corrected MML estimate where all the values are censored to the lowest value in a given data set.

By taking into account the possibility that a single minimum value in a data set can become significant (i.e. capable of triggering brittle failure) due to severe local microstructural inhomogeneity of the material, the procedure can be applied in the cases where the weld heat-affected zone of an otherwise tough steel exhibits local brittle zones.

In the last stage of the procedure, the final \bar{K}_{MAT} fracture toughness estimate (and its probability distribution) are calculated using the estimate obtained either according to Step 1, 2 or 3. This information can then be subsequently applied to structural integrity assessments.

5.4.1 Data at a single temperature

In the case of data at a single temperature, two fracture toughness parameters: K_0 and \bar{K}_{MAT} are calculated, similarly to Step 1 and Step 2, see Sections 5.2.1 and 5.3.1.

Of these two, K_0 corresponding to a 63.2 % cumulative failure probability K_{MAT} is calculated according to Eq. (6a) as:

$$K_0 = 20 \text{ MPa}\sqrt{m} + (K_{MAT_{min}} - 20 \text{ MPa}\sqrt{m}) \cdot \left(\frac{N}{\ln 2} \right)^{1/4} \dots\dots\dots(6a)$$

where $K_{MAT_{min}}$ is a minimum (lowest) individual K_{MAT} value in a given data set and N is the total number of tests in the set.

It is seen that Eq. (6a) is basically similar to Eq. (5a), with the exception of now using the minimum individual K_{MAT} value, $K_{MAT_{min}}$, instead of a number of different individual values (K_{MAT_i}). Parameter N, in turn, takes into account the influence of the total number of results in a given data set on K_0 estimate, increasing number of tests having a positive influence on K_0 .

Using K_0 obtained from Eq. (6a), \bar{K}_{MAT} corresponding to the 50 % failure probability is calculated according to Eq. (3), similarly to that according to Step 1 and 2 in Sections 5.2 and 5.3.

In the next stage, the \bar{K}_{MAT} (K_0) estimates according to Steps 1, 2 and 3 are compared with each other, in order to obtain a final characteristic toughness estimate.

Provided that the obtained fracture toughness K_0 according to Step 3 and calculated from Eq. (6a) is more than 10 % lower than the corresponding fracture toughness according to Eq. (5a) in Step 1 and Step 2 - whichever of them is lower, i.e. $K_{0step3} < 0.9 K_{0step1\&2}$, this single minimum test result in a given data set that was used to calculate K_0 is regarded as significant. Consequently, the Step 3 estimate for K_0 is taken as a final estimate of material's fracture toughness, \bar{K}_{MAT} , for further calculations, together with its probability distribution $P\{K_{MAT}\}$ in the next stage of the Procedure according to Section 5.5.

In the case that the fracture toughness K_0 according to Step 3 and calculated from Eq. (6a) remains equal to or less than 10 % below the fracture toughness according to Eq. (5b) in Step 1 and Step 2, this single minimum test result in the data set is considered as non-significant. Consequently, the lower one of the Step 1 and Step 2 estimates for K_0 is taken for calculating the final fracture toughness estimate, i.e. \bar{K}_{MAT} , together with its probability distribution $P\{K_{MAT}\}$ according to Section 5.5.

5.4.2 Data at different temperatures

In the case of data at different temperatures, transition temperature T_0 corresponding to the 100 MPa \sqrt{m} median (50 %) K_{MAT} transition temperature is taken as a maximum of the individual values calculated according to Eq. (6b) as:

$$T_0 = \max_i T_i - \frac{\ln \left\{ \frac{(K_{MAT_i} - 20 \text{ MPa}\sqrt{m}) \cdot \left(\frac{N}{\ln 2}\right)^{1/4} - 11 \text{ MPa}\sqrt{m}}{77} \right\}}{0.019} \quad (d_i = 1) \quad \dots\dots\dots(6b)$$

where T_i is individual transition temperature of a specimen of toughness K_{MAT_i} , K_{MAT_i} is individual fracture toughness of a specimen (size-adjusted according to Eq. (1) whenever necessary) and N is the total number of tests. Note that the expression assumes the use of only brittle test results by designating $\delta_i = 1$.

In the next stage, the $T_0(\bar{K}_{MAT})$ estimates according to Steps 1, 2 and 3 are compared with each other, in order to obtain a final characteristic toughness estimate.

Provided that the transition temperature T_0 according to Step 3 and calculated from Eq. (6b) is more than 8 °C above the transition temperature according to Eq. (5b) in Step 1 and Step 2 - whichever of them is higher, i.e. $T_{0step3} > T_{0step1\&2} + 8 \text{ °C}$, this single minimum test result in a given data set that was used to calculate T_0 is considered as significant. Consequently, the Step 3 estimate for T_0 is taken as a final estimate of material's fracture toughness, \bar{K}_{MAT} , for further calculations, together with its probability distribution $P\{K_{MAT}\}$ in the next stage of the Procedure according to Section 5.5.

In the case that the transition temperature T_0 according to Step 3 and calculated from Eq. (6b) remains equal to or less than 8 °C above the transition temperature according to Eq. (5b) in Step 1 and Step 2, this single minimum test result in the data set is considered as non-significant. Consequently, the higher one of the Step 1 and Step 2 estimates for T_0 is taken for calculating the final fracture toughness estimate, i.e. \bar{K}_{MAT} , together with its probability distribution $P\{K_{MAT}\}$ according to Section 5.5.

5.5 Determination of final K_{MAT} estimate (\bar{K}_{MAT}) and probability distribution $P\{K_{MAT}\}$

In the last stage of the Procedure, the final \bar{K}_{MAT} fracture toughness estimate, together with its probability distribution $P\{K_{MAT}\}$ is calculated. For this, the calculation procedure uses the estimates obtained either according to Step 1, 2 or 3 - i.e. $T_{0(step1-3)}$ or $K_{0(step1-3)}$ - and which hence are chosen according to the criteria given in the context of each step.

5.5.1 Additional safety factors for median \bar{K}_{MAT} estimate for small number of tests

To account for the cases where the number of tests is very small considering the statistical treatment of the fracture toughness test data, additional safety factors for K_0 and T_0 are introduced, with the intention of penalising the limited amount of data.

For data at a single temperature, the safety corrected K_0 is calculated according to Eq. (11), as:

$$K_0 = K_{min} + \frac{\hat{K}_0 - K_{min}}{\left(1 + \frac{0.25}{\sqrt{r}}\right)} \quad \dots\dots\dots(11)$$

For data at different temperatures, the safety corrected T_0 is calculated according to Eq. (12), as:

$$T_0 = \hat{T}_0 + \left(\frac{140C}{\sqrt{r}} \right) \dots\dots\dots(12)$$

where: r is the number of toughness values corresponding to brittle failure, and $\frac{\hat{K}_0}{\left(\frac{K_0}{1.7}\right)}$ and \hat{T}_0 are the original values for fracture toughness (in the case of data at a single temperature) and fracture toughness transition temperature (in the case of data at different temperatures), respectively.

The use of these safety corrected values of K_0 and T_0 as an input to Eqs (3) and (4b), respectively, guarantees that the desired median K_{MAT} fracture toughness estimate (\bar{K}_{MAT}) is obtained with a confidence of 85 % ($P_{conf} = 85\%$).

With an increasing number of tests, the influence of the corrections according to Eqs (11) and (12) on the final fracture toughness estimate is gradually reduced, and finally approaches to nil.

5.5.2 \bar{K}_{MAT} for data at a single temperature

In the case of data at a single temperature, the median K_{MAT} fracture toughness estimate (\bar{K}_{MAT}) is simply calculated according to Eq. (3) by using $K_{0(step1-3)}$ as an input value, see sections 5.2.1 - 5.2.3, as:

$$\bar{K}_{MAT} = 20 MPa\sqrt{m} + (K_0 - 20 MPa\sqrt{m}).0.91 \dots\dots\dots(3)$$

5.5.3 \bar{K}_{MAT} for data at different temperatures

In the case of data at different temperatures, the median K_{MAT} fracture toughness estimate (\bar{K}_{MAT}) is calculated according to Eq. (4b) by using $T_{0(step1-3)}$ as an input value, as:

$$\bar{K}_{MAT} = 30 + 70.\exp\{0.019.(T - T_0)\} \dots\dots\dots(4b)$$

Alternatively, \bar{K}_{MAT} can also be approximated by using Eq. (4a) and Eq. (3) in combination, as:

$$K_0 \approx 31 + 77.\exp\{0.019.(T - T_0)\} \dots\dots\dots(4a)$$

$$\bar{K}_{MAT} = 20 MPa\sqrt{m} + (K_0 - 20 MPa\sqrt{m}).0.91 \dots\dots\dots(3)$$

5.5.4 Probability distribution of K_{MAT} estimate

The probability distribution corresponding to the median K_{MAT} estimate, $P\{K_{MAT}\}$, is calculated according to Eq. (2) as:

$$P\{K_{MAT}\} = 1 - \exp - \left\{ \frac{K_{MAT} - 20 MPa\sqrt{m}}{K_0 - 20 MPa\sqrt{m}} \right\}^4 \dots\dots\dots(2)$$

5.5.5 Size re-adjustment of \bar{K}_{MAT} estimate

For the structural integrity assessment, the hereby obtained median K_{MAT} estimate corresponding to 'normalisation' fracture toughness (B = 25 mm), K_{MAT25} , may be size re-adjusted to correspond to an estimate related to any required crack size (or specimen size) by using Eq. (1) given in Section 5.1.

6. ADDITIONAL GUIDANCE FOR USING THE PROCEDURE

To ensure the reliable application of the procedure for both base materials and welded joints, the user must realise a few essential premises regarding the fracture mechanics testing practice, as well as the particular application whose structural integrity is going to be assessed.

Particular attention should be placed on ensuring that the data set is representative to the application of the structure/component being assessed. Priority should be given to the use of material's data obtained at a test temperature range corresponding to the intended operating temperature of the actual structure / component. For the cases where only ductile fracture data are available, but the possibility of brittle fracture in a structure cannot be excluded, Step 3 of the procedure, which treats the minimum initiation value as a brittle cleavage fracture event, can be reliably used for fracture toughness estimation in the case of macroscopically homogeneous material. Contrary to this, in the cases where the operating temperature of a structure is close to the material's upper shelf, but only brittle fracture data are available, it is advisable to generate appropriate ductile fracture data since a material's upper-shelf behaviour cannot be reliably predicted from brittle fracture data alone.

Special attention should be placed on ensuring that microstructurally representative data is available in the case of welded components. In the case of welded joints, data should be available for all the 'critical' weld regions, such as the coarse-grained HAZ and the weld metal. No matter how advanced the procedure itself, it cannot be expected to yield reliable prediction in the cases where data from the most critical region of the welded joint, e.g. CGHAZ, would be totally absent. With respect to the test performance practices, guidance can be found from relevant test standards, e.g. BS 7448 Part 2, which can be followed regarding aspects such as notch placement, crack front straightness, minimum fatigue crack length and post-test sectioning of the specimens. *It is important, however, the validity of a single fracture mechanics test result not to be judged by any microstructural criterion, such as the minimum 15 % (or 20 %) coarse-grained HAZ required for a valid test etc.,* because this may lead to the rejection of relevant data from the SINTAP procedure viewpoint, with subsequent distortion of the data set to be used for the assessment.

Definition of the MMI censoring levels and classification of fracture toughness values for censoring. Censoring levels are defined according to ASTM E1921-97 for K values. The classification (brittle vs. ductile) should be made according to the *final failure mode* of the specimen and the associated fracture toughness value designated either as: $\delta = 1$ (brittle failure) or $\delta = 0$ (censored). Thus, initiation of tearing does not automatically require censoring, unless the final failure occurs in a ductile manner. In the case where more than one fracture toughness parameter can be determined from the same test, designation of the fracture toughness value should be made according to the *failure mechanism associated with the particular value used* for the analysis.

For the final structural integrity assessment, suitable confidence and probability levels should be chosen in relation to the criticality of the particular component/structural member being assessed. The 'amount of safety' of a structure or component should depend on its particular application conditions. This means that selecting a confidence level to be used in the structural integrity assessment depends on the criticality of the component/structural member in question, not the quality of the data used for toughness estimation. For the assessment, suitable confidence and probability levels should hence be chosen in terms of the final application. Therefore, for very critical structural parts, a more conservative confidence level is advised to be chosen, irrespective of the quality of the data itself that was used for material's fracture toughness assessment stage for obtaining K_{mat} estimate.

7. TREATMENT OF DUCTILE FRACTURE DATA

Even in the case that ductile fracture data is all that is available for the assessment, the philosophy of design of a structure can, apart from the type of the available data, be based either on "design against brittle fracture" or "design against ductile fracture". For structures with their operating temperature in the transition regime or close to the lower shelf, the recommended practice is the former, in the case which the approach is, again, based on the "Master Curve" prediction but, now, using the minimum data treated as brittle.

For ferritic steels on the upper shelf of fracture toughness, or for materials which do not exhibit brittle cleavage fracture, design against brittle fracture is not necessary. For these cases, a separate approach is presented to conduct the correct treatment of data for obtaining a reliable fracture toughness estimate.

7.1 General characteristics of ductile fracture

The ductile fracture process which consists of micro-void formation, growth and coalescence, is usually characterised, in terms of J integral or crack opening displacement, with the associated amount of stable crack extension, Δa . Consequently, a single value of fracture toughness is not appropriate. The fracture resistance curve, J - Δa say, is often measured in so-called single specimen tests with the crack extension Δa deduced by an indirect method such as the elastic unloading compliance method or by electrical potential drop techniques.

Ductile crack growth has usually a much smaller scatter than brittle cleavage fracture. The main source of uncertainty and scatter in ductile fracture is the test performance and data analysis, not the behaviour or macroscopical inhomogeneity of the material. This is true especially for single specimen tests.

Here it is assumed that, irrespective of the type of the original toughness data available, it has been converted into a stress intensity factor equivalent, K_{mat} . However, in contrast to Sections 3-5, a single value of K_{mat} with an associated probability distribution is not defined. Instead, it is assumed that values of K_{mat} are available at an engineering definition of initiation and at a number of values of crack growth, Δa . Initiation may be determined according to a recognised standard and K_{mat} would then be equal to $K_{0.2}$ or $K_{0.2/BL}$, say, as defined by the standard. Note that subsequent data are assumed to be defined by values $K_{mat}(\Delta a)$ at specified amounts of crack extension rather than by the slope of a resistance curve. Bounds to the latter in conjunction with bounds to an initiation value can lead to unrepresentative results when there are correlations between initiation toughness and the slope of the resistance curve. For values beyond initiation, therefore, $K_{mat}(\Delta a)$ must be evaluated from the test specimen data at the same amount of crack growth in each test.

7.2 Approaches for treatment of ductile fracture data

The approach adopted for the treatment of ductile fracture data depends on the design philosophy of a structure, as well as the number of specimens tested.

7.2.1 Design against brittle fracture

If the possibility of brittle cleavage fracture in the structure cannot be excluded, the minimum initiation value should be ~~treated~~ as a brittle cleavage fracture event. Consequently, brittle fracture analysis should be performed according to the Step 3 *Minimum Value Estimation* of the 'SINTAP' Procedure presented in Section 5.4. This guarantees a reliable fracture toughness estimate in the case of macroscopically homogeneous material.

7.2.2 Design against ductile fracture

For ferritic steels (i) on the upper shelf of fracture toughness, or for (ii) materials which do not exhibit brittle cleavage fracture, design against brittle fracture is not necessary. Therefore, a different approach must be chosen. Due to insufficient knowledge of the extent to which all the factors (e.g. constraint, mismatch, scatter, definition of "initiation", testing, etc.) influencing the ductile fracture behaviour should actually be taken into account in the prediction, this approach is considered to be informative only.

The approach consists of three different levels depending on the type of data that is available in each case.

If only Charpy data are available, the *lower bound Charpy correlation* should be used according to SINTAP Default Procedure Appendix 1.

The use of direct ductile tearing data is permitted provided that at least three (3) normally identical fracture toughness test results are available. For the limited number of available test results between 3 to 9, *the minimum J-R curve* may be used to obtain a *lower bound estimate* to the fracture toughness.

For 10 or more tests, a *full statistical analysis* that assumes normally distributed absolute scatter in J is permitted and may hence be used. The equivalent K_{mat} value is then calculated from J according to the conversions in Section 3.3.

The scatter in ductile fracture toughness data has been found to be broadly independent of the level of crack extension. Therefore, the standard deviation at one value of Δa , 1 mm say, may be used to estimate scatter at other values of K_{mat} . The expected scatter in the J-R curve is known to be best described as a normally distributed absolute scatter in J-integral values. For a range of materials, the scatter has been found to be typically less than 10 % of the mean value corresponding to a crack extension of 1 mm. The standard deviation therefore is conservatively described as 10 % of the mean J-integral corresponding to a crack growth of 1 mm.

For the statistical analysis of ductile fracture data, the mean toughness (\bar{K}_{MAT}) is simply given as:

$$\bar{K}_{mat} = \frac{\sum_{i=1}^n K_{mat\ j}}{n} \dots\dots\dots(7)$$

where n is the number of individual $K_{mat,i}$ data points, at initiation or for a specific value of Δa .

To evaluate the distribution of toughness, the variance to the data (S) can be defined as:

$$S^2 = \frac{\sum_{i=1}^n (K_{mat\ j} - \bar{K}_{mat})^2}{(n - 1)} \dots\dots\dots(8)$$

where n is the number of individual $K_{mat,i}$ data points, at initiation or for a specific value of Δa , $K_{mat,i}$ is an individual data point and \bar{K}_{MAT} is the mean toughness.

Confidence limits to the data (K_{mat^∞}) are then given by:

$$K_{mat\ \infty} = \bar{K}_{mat} \pm t_\infty S \dots\dots\dots(9)$$

where ∞ is the confidence limit and t_∞ is the corresponding value of Student's distribution at (n - 1) degrees of freedom.

In some cases, confidence limits to the mean of the data are required. These are obtained in a similar manner to Eq. (9) as:

$$\bar{K}_{mat\ \infty} = \bar{K}_{mat} \pm t_\infty S / \sqrt{n} \dots\dots\dots(10)$$

Usually, wide bounds to the data are obtained when only few specimens have been tested. While more vigorous statistical treatments of data may be attempted, it is preferable to test more specimens to increase confidence.